Lesson 3: MHD reconnection, MHD currents

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MHD basics

- MHD cannot address discrete or single particle effects such as gyro motion and small-scale effects (smaller than the ion gyroradius r_i).
- MHD equations are valid for much lower frequencies than the plasma frequency ω_{pe}.
- In Maxwell's equations the displacement current $\epsilon_0 \partial \mathbf{E}/\partial t$ has been neglected by assuming that there are no electromagnetic waves propagating at the speed of light.

Assuming electrons and single charged ions and neutral plasma $n_e = n_i = n$, the total current density $\mathbf{j}$, the total mass density ρ_m, effective mass M, and total mass velocity flux $\rho_m \mathbf{v}$ are given by

$$\mathbf{j} = en(\mathbf{v}_i - \mathbf{v}_e) \quad (5.17)$$

$$\rho_m = n(m_i + m_e) \quad (5.18)$$

$$M = m_i + m_e \quad (5.19)$$

$$\rho_m \mathbf{v} = n(m_i \mathbf{v}_i + m_e \mathbf{v}_e) \quad (5.20)$$

MHD equations

One-fluid resistive MHD equations:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}) = 0 \qquad \text{(mass continuity equation)} \qquad (5.26)$$

$$\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) \rho_m = \mathbf{j} \times \mathbf{B} - \nabla p \qquad \text{(momentum equation)} \qquad (5.27)$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{\mathbf{j}}{\sigma} \qquad \text{(generalized Ohm's law)} \qquad (5.28)$$

$$\frac{d}{dt}(p \rho_m^{-\gamma}) = 0 \qquad \text{(equation of state)} \qquad (5.29)$$

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} \qquad \text{(Ampère's law)} \qquad (5.30)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (5.31)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{(Faraday's law)} \qquad (5.32)$$

$$\nabla \cdot \mathbf{E} = 0 \qquad \text{(Gauss' law)} \qquad (5.33)$$

When plasma conductivity $\sigma = ne^2/m_e \nu_e \rightarrow \infty$, (5.28) => Ideal MHD

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \quad \Rightarrow \quad \mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

Induction equation

Let's start from the resistive MHD Ohm's law in eq. (5.28)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{\mathbf{j}}{\sigma} \qquad (5.35)$$

and take the curl and use Faraday's law in eq. (5.32) to get

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \frac{\mathbf{j}}{\sigma}) \ . \qquad (5.36)$$

We insert $\mathbf{j}$ from Ampère's law in eq. (5.30) and use the identity (see Appendix A)

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} \qquad (5.37)$$

together with eq. (5.31). The result is the *induction equation*

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \ .} \qquad (5.38)$$

convection diffusion

Convection

If $\sigma \rightarrow \infty$ in eq. (5.38), we get the convection equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

If plasma moves, the magnetic field lines must follow, because they can't diffuse across plasma. It is said that the magnetic is *frozen in the plasma*.

By applying Faraday's law on the left hand side of the eq., we immediately see that

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

which gives an equation for plasma velocity

$$\mathbf{v} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \ .$$

This approximation is valid in most parts of the magnetosphere, but can be violated e.g. at boundaries and in the reconnection (magnetic merging) regions. Even though conductivity is not infinite in the ionosphere, collisions are infrequent in the F region and the equation above is valid also for plasma in the F region.

Diffusion

If plasma is at rest (**v** = 0), the induction equation simplifies to diffusion equation

$$\frac{\partial \mathbf{B}}{\partial t} = D_m \nabla^2 \mathbf{B} \ ,$$

where the diffusion coefficient D_m is

$$D_m = \frac{1}{\mu_0 \sigma} \ .$$

The characteristic time of magnetic diffusion is found by replacing ∇^2 by $1/L_B^2$, where L_B is the characteristic gradient length of the inhomogeneity in the magnetic field. Then B can be solved as

$$B \ = B_0 \exp(\pm t/\tau_d) \ .$$

where the magnetic diffusion time τ_d is given by

$$\tau_d = \mu_0 \sigma L_B^2 \ = L_B^2 / D_m .$$

If $\sigma \rightarrow \infty$ (or L_B is very large), the diffusion time becomes very long and magnetic field is not able to diffuse across plasma.

Magnetic merging

If the magnetic induction eq. (5.38) is written in a simple dimensional form

$$\frac{B}{\tau} = \frac{vB}{L_B} + \frac{B}{\tau_d} \ . \tag{5.46}$$

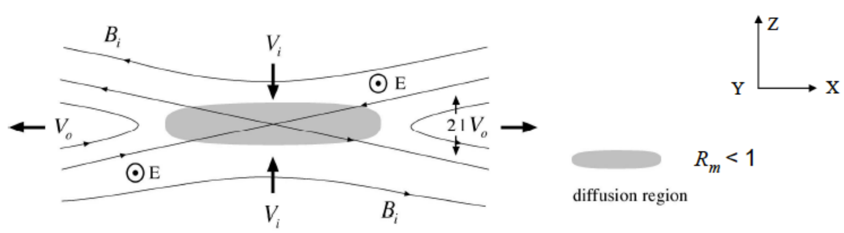
The ratio of the first and second term give the *magnetic Reynolds number*

$$R_m = \mu_0 \sigma L_B v \ . \tag{5.47}$$

If $R_m \gg 1$ convection dominates and diffusion can be neglected. For example, the solar wind magnetic Reynolds number is about $R_m \approx 7 \cdot 10^{16}$.

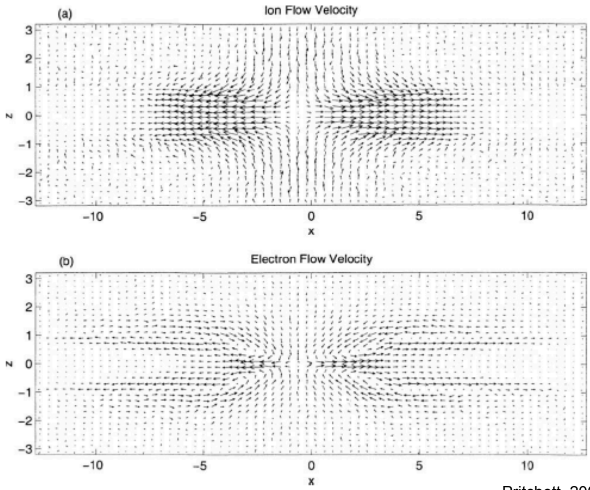
However, if plasma velocity v or the gradient scale length L_B or conductivity σ decreases, magnetic field starts to diffuse. This may occur within a very limited region, e.g. at the subsolar magnetopause or in the magnetotail.

X-type neutral line



- Magnetic field diffusion (R_m < 1) occurs in a limited region.
- Magnetic field is zero only at a single line, at the neutral line (in Y-direction).
- Constant E_y (reconnection electric field) in steady-state.

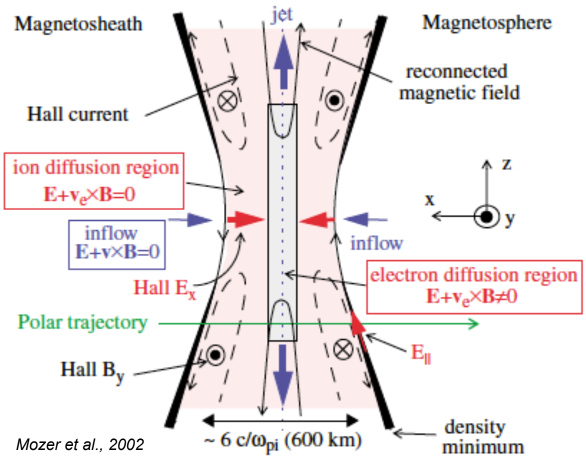
X-type neutral line: reality is more complicated



Pritchett, 2001

When the diffusion region width becomes smaller than the ion inertial length $\delta_i = c/\omega_{pi}$, ions start to diffuse from the magnetic field (top panel), whereas electrons still follow the ExB-drift (bottom panel).

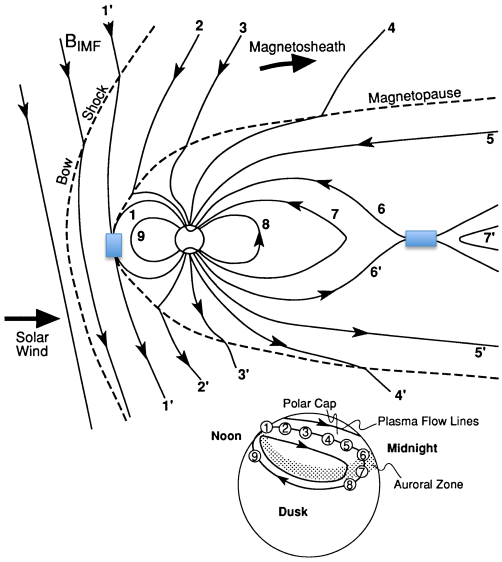
X-type neutral line: reality is more complicated



Mozer et al., 2002

Ions diffuse from the magnetic field in the *ion diffusion region*, whereas magnetic field remains frozen into the motion of the electrons and diffuse later inside the smaller *electron diffusion* region. Separation of ions and electrons sets up *the Hall current system*.

Plasma convection



Plasma convection in the ionosphere is due to reconnection of the IMF and the geomagnetic field at the dayside magnetopase and in the magnetotail.

The result is the 2-cell convection pattern in the ionosphere during southward IMF conditions.

During northward IMF, reconnection may take place poleward of the cusp.

MHD perpendicular currents

We start from the momentum equation (5.27) and take the cross product with $\mathbf{B}$

$$\mathbf{j}_\perp = -\frac{1}{B^2}\left(\rho_m \frac{d\mathbf{v}}{dt} \times \mathbf{B} + \nabla p \times \mathbf{B}\right) \quad . \qquad (5.62)$$

The second term gives the *diamagnetic current* (perpendicular to **B**)

$$\mathbf{j}_\perp = \frac{\mathbf{B} \times \nabla p}{B^2}$$

and the first term gives the *polarization current*, aka *inertial current*, (also perpendicular to **B**). The second form comes by using **E**=-**v** x **B**.

$$\mathbf{j}_\perp = -\frac{\rho_m}{B^2}\frac{d\mathbf{v}}{dt} \times \mathbf{B} \quad \Leftrightarrow \quad \mathbf{j}_\perp = \frac{\rho_m}{B^2}\frac{\partial \mathbf{E}_\perp}{\partial t}$$

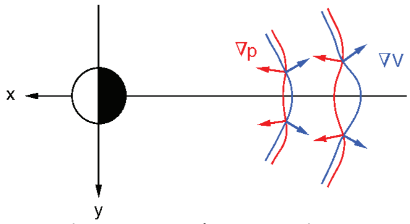
MHD FAC from diamagnetic current

By using the condition of current continuity, $\nabla \cdot \mathbf{j}_{\|} = -\nabla \cdot \mathbf{j}_{\perp}$ it can be shown (Vasyliunas, 1970) that we get

$$\int_{eq}^{ion} \frac{j_{\|}}{B} = -\frac{\mathbf{B}_{eq}}{B_{eq}^2} \cdot \nabla p_{eq} \times \nabla V \ , \qquad (5.65)$$

the so called Vasyliunas equation. Here V is the differential flux tube volume (i.e. the volume of a magnetic flux tube of unit magnetic flux). This volume is given by

$$V = \int_{eq}^{ion} \frac{ds}{B} \ , \qquad (5.66)$$



Diversion of the diamagnetic current at the magnetic equator plane to produce FAC.

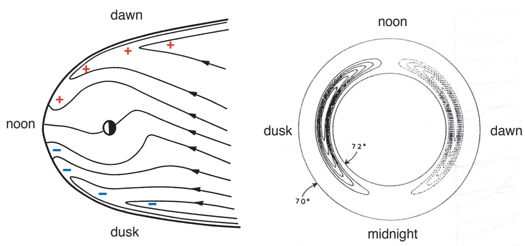
MHD FAC from the inertial current

Similarly, it can be shown that for the inertial current we get the following FAC

$$\int_{eq}^{ion} \frac{j_{\|}}{B} = -\int_{eq}^{ion} \frac{\rho_m}{B^2} \frac{d\Omega_{\|}}{dt} ds$$

where the field-aligned component of plasma vorticity is given by

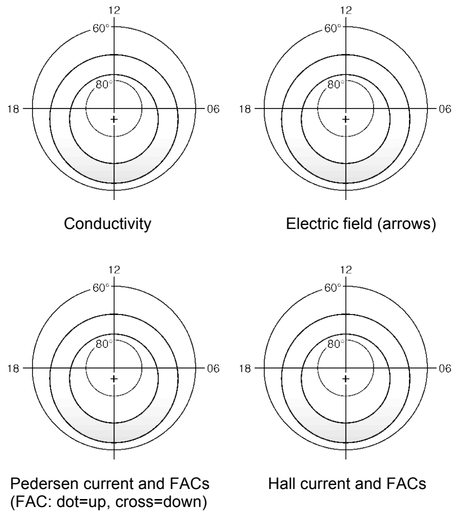
$$\mathbf{\Omega} = \nabla \times \mathbf{v} \quad \Rightarrow \quad \Omega_{\|} = \hat{\mathbf{b}} \cdot \nabla \times \ \mathbf{v}$$



Plasma flow in the eq. plane (left) and FACs by vorticity (right): solid lines at dusk correspond to upward FAC from the ionsophere and dashed lines in the dawn to downward FAC.

Group task 3: What kind of horizontal and F-A currents are flowing in the polar ionosphere?

Exercise: Add EF and currents in the three panels.



Group task 3: What kind of horizontal and F-A currents are flowing in the polar ionosphere?

Solution on the blackboard.