

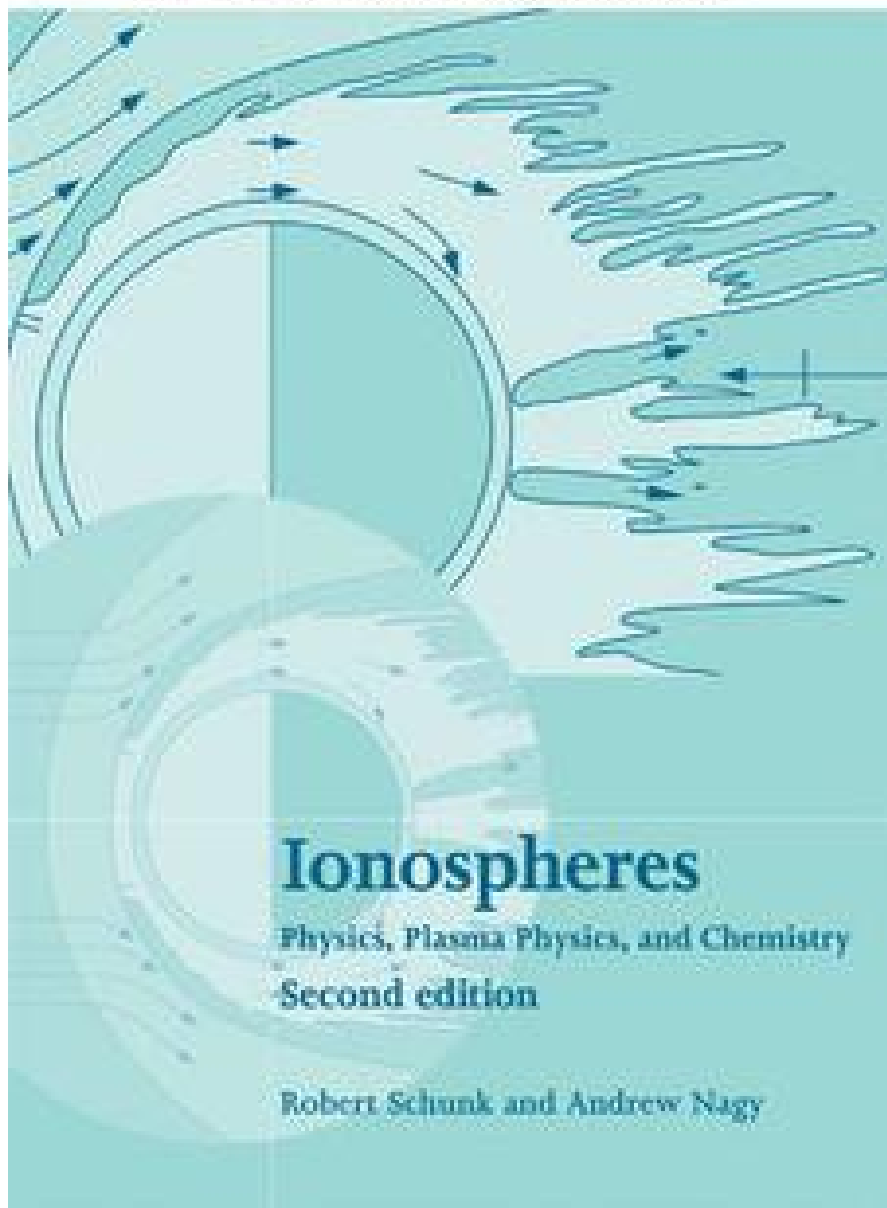
# Auroral aeronomy

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AGF-351

Optical methods in auroral physics research,  
UNIS, Longyearbyen

Cambridge Atmospheric and Space Science Series



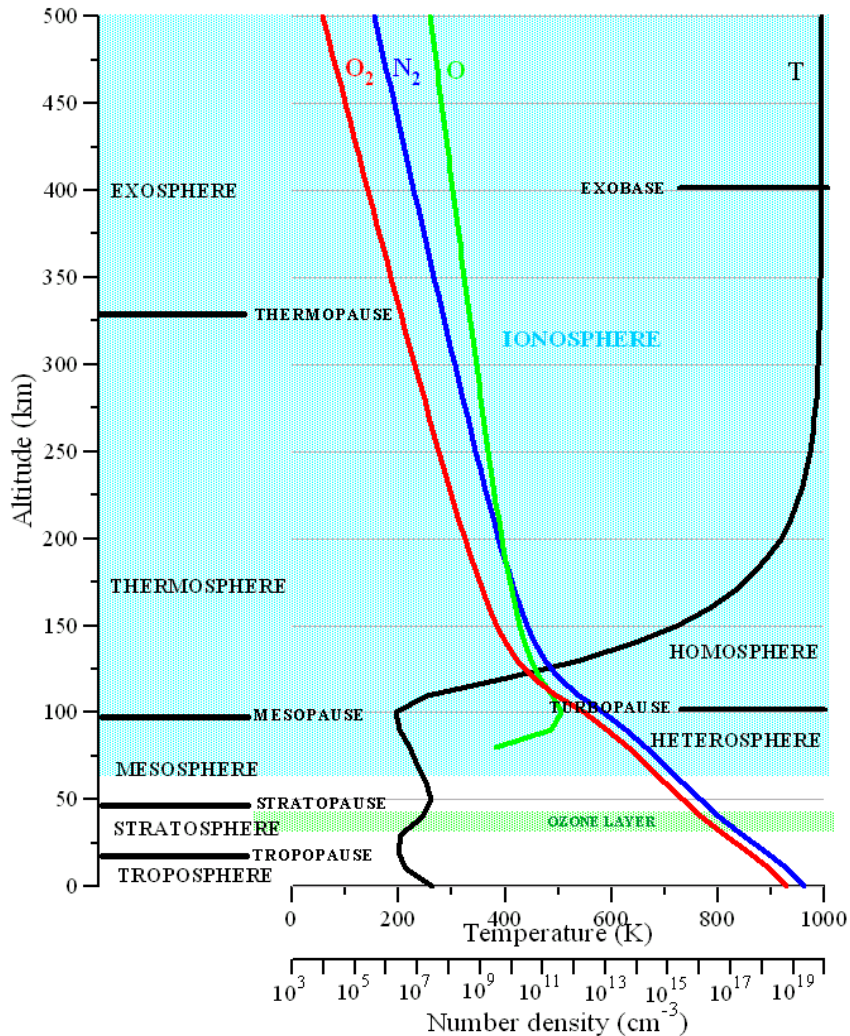
# Ionospheres

Physics, Plasma Physics, and Chemistry

Second edition

Robert Schunk and Andrew Nagy

# The Earth's atmosphere



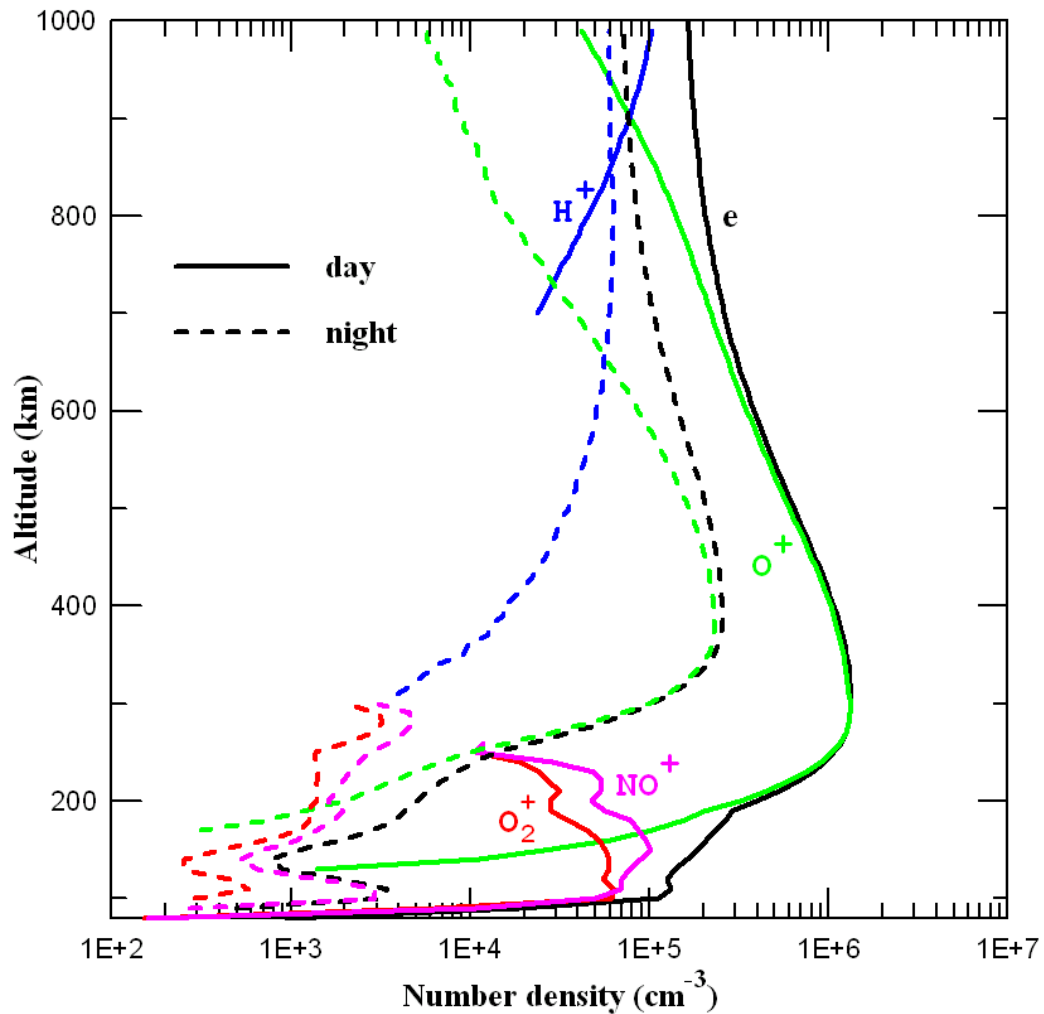
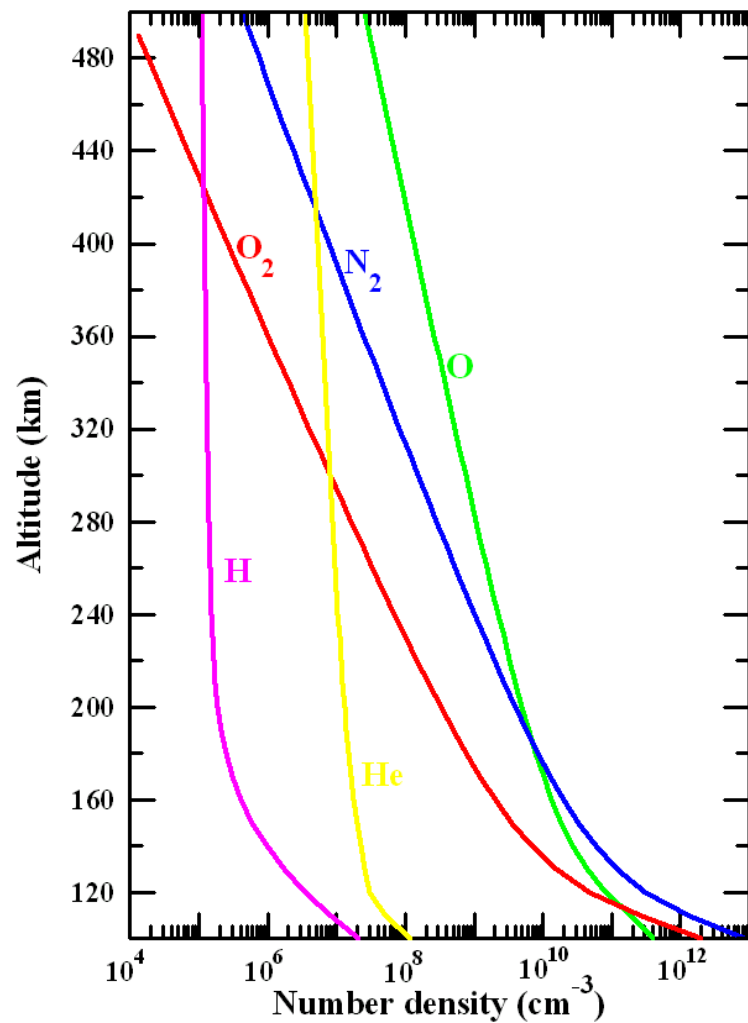
## Temperature stratified layers

- Troposphere: ~0-10 km
- Stratosphere: ~10-45 km
- Mesosphere: ~40-95 km
- Thermosphere: ~95-400 km

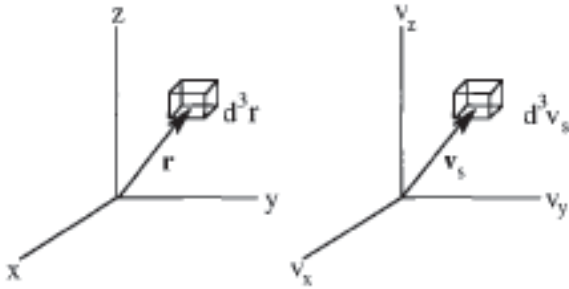
## Moving stratified layers

- Heterosphere: ~0-100 km turbulent moving, the atmospheric species are mixed.
- Homosphere: ~100-400 km molecular diffusion, diffusion separation of the various species
- Exosphere: >400 km the neutral particles follows ballistic trajectories.

# Earth's thermosphere and ionosphere



## Distribution function and Boltzmann equation



Distribution function  $f_s(\mathbf{r}, \mathbf{v}_s, t)$  corresponds to the number of particles of species  $s$  that, at time  $t$ , are located in a volume element  $d^3r$  about  $\mathbf{r}$  have velocities in a velocity-space volume element  $d^3v_s$  about  $\mathbf{v}_s$ . Alternatively,  $f_s$  can be viewed as a probability density in the  $(\mathbf{r}, \mathbf{v}_s)$  phase space.

$$\frac{df_s(\mathbf{r}, \mathbf{v}_s, t)}{dt} = \frac{\partial f_s}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \nabla f_s + \frac{d\mathbf{v}_s}{dt} \cdot \nabla_{\mathbf{v}_s} f_s$$

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \nabla f_s + \mathbf{a}_s \cdot \nabla_{\mathbf{v}_s} f_s = 0$$

Vlasov equation

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \nabla f_s + \mathbf{a}_s \cdot \nabla_{\mathbf{v}_s} f_s = \frac{\delta f_s}{\delta t}$$

Boltzmann equation

$$\frac{\delta f_s}{\delta t} = \iint d^3v_t d\Omega |\mathbf{v}_s - \mathbf{v}_t| \sigma_{st}(\mathbf{v}_{st}, \theta) (f'_s f'_t - f_s f_t) \quad \text{Boltzmann collision integral}$$

$$\mathbf{a}_s = \mathbf{G} + \frac{e_s}{m_s} (\mathbf{E} + \mathbf{v}_s \times \mathbf{B})$$

Gravitational and Lorentz accelerations

## Moments of the distribution function

$$\langle \xi(\mathbf{v}_s) \rangle = \int d^3v_s f_s(r, \mathbf{v}_s, t) \xi(\mathbf{v}_s)$$

Zero moment	$n_s = \int d^3v_s f_s$	Number density
First moment	$\mathbf{u}_s = \frac{1}{n_s} \int d^3v_s \mathbf{v}_s f_s$	Drift velocity
Second moment	$T_s = \frac{3m_s}{k} \int d^3v_s (\mathbf{v}_s - \mathbf{u}_s)^2 f_s$	Temperature
Third moment	$\mathbf{q}_s = \frac{m_s}{2} \int d^3v_s (\mathbf{v}_s - \mathbf{u}_s)^2 (\mathbf{v}_s - \mathbf{u}_s) f_s$	Heat flow

## General transport equations

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \nabla f_s + \mathbf{a}_s \cdot \nabla_{\mathbf{v}} f_s = \frac{\delta f_s}{\delta t}$$

$$\nabla \cdot (f_s \mathbf{a}_s) = \mathbf{a}_s \cdot \nabla f_s + f_s (\nabla \cdot \mathbf{a}_s) = \mathbf{a}_s \cdot \nabla f_s$$

$$\nabla \cdot (f_s \mathbf{v}_s) = \mathbf{v}_s \cdot \nabla f_s + f_s (\nabla \cdot \mathbf{v}_s) = \mathbf{v}_s \cdot \nabla f_s$$

$$\frac{\partial f_s}{\partial t} + \nabla \cdot (f_s \mathbf{v}_s) + \nabla \cdot (f_s \mathbf{a}_s) = \frac{\delta f_s}{\delta t}$$

$$\int d^3 v_s \frac{\partial f_s}{\partial t} = \frac{\partial}{\partial t} \int d^3 v_s f_s = \frac{\partial n_s}{\partial t} \quad \int d^3 v_s \nabla \cdot (f_s \mathbf{v}_s) = \nabla \cdot \int d^3 v_s f_s \mathbf{v}_s = \nabla \cdot (n_s \mathbf{u}_s)$$

$$\int d^3 v_s \nabla \cdot (f_s \mathbf{a}_s) = \int_S dA_{\mathbf{v}} (f_s \mathbf{a}_s) \cdot \hat{\mathbf{n}}_{\mathbf{v}} = 0 \quad \int d^3 v_s \frac{\delta f_s}{\delta t} = \frac{\delta n_s}{\delta t}$$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{u}_s) = \frac{\delta n_s}{\delta t}$$

Continuity equation

## 13-moment transport equations

Continuity equation

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{u}_s) = \frac{\delta n_s}{\delta t}$$

Momentum equation

$$n_s m_s \left( \frac{\partial \mathbf{u}_s}{\partial t} + \mathbf{u}_s \cdot \nabla \cdot \mathbf{u}_s \right) + \nabla p_s + \nabla \cdot \boldsymbol{\tau}_s - n_s m_s \mathbf{g} - n_s e_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) = \frac{\delta \mathbf{M}}{\delta t}$$

Energy equation

$$\frac{3}{2} \left( \frac{\partial p_s}{\partial t} + \mathbf{u}_s \cdot \nabla p_s \right) + \frac{3}{2} p_s (\nabla \cdot \mathbf{u}_s) + \nabla \cdot \mathbf{q}_s + \boldsymbol{\tau}_s : \nabla \mathbf{u}_s = \frac{\delta E_s}{\delta t}$$

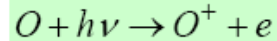
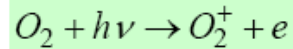
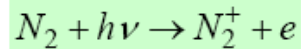
Pressure tensor equation

$$\begin{aligned} & \frac{\partial \boldsymbol{\tau}_s}{\partial t} + \mathbf{u}_s \cdot \nabla \boldsymbol{\tau}_s + \boldsymbol{\tau}_s (\nabla \cdot \mathbf{u}_s) + \frac{e_s}{m_s} [\mathbf{B} \times \boldsymbol{\tau}_s - \boldsymbol{\tau}_s \times \mathbf{B}] + p_s \left[ \nabla \mathbf{u}_s + (\nabla \mathbf{u}_s)^T - \frac{2}{3} (\nabla \cdot \mathbf{u}_s) \mathbf{I} \right] + \\ & + \frac{2}{5} \left[ \nabla \mathbf{q}_s + (\nabla \mathbf{q}_s)^T - \frac{2}{3} (\nabla \cdot \mathbf{q}_s) \mathbf{I} \right] + \left[ \boldsymbol{\tau}_s \cdot \nabla \mathbf{u}_s + (\boldsymbol{\tau}_s \cdot \nabla \mathbf{u}_s)^T - \frac{2}{3} (\boldsymbol{\tau}_s : \nabla \mathbf{u}_s) \mathbf{I} \right] = \frac{\delta \boldsymbol{\tau}_s}{\delta t} \end{aligned}$$

Heat flow equation

$$\begin{aligned} & \frac{\partial \mathbf{q}_s}{\partial t} + \mathbf{u}_s \cdot \nabla \mathbf{q}_s + \frac{7}{5} \mathbf{q}_s \cdot \nabla \mathbf{u}_s + \frac{7}{5} \mathbf{q}_s (\nabla \cdot \mathbf{u}_s) + \frac{5}{2} (\nabla \mathbf{u}_s) \cdot \mathbf{q}_s + \frac{5 k p_s}{2 m_s} \nabla T_s + \frac{1}{n_s m_s} (\nabla \cdot \boldsymbol{\tau}_s) \cdot (p_s \mathbf{I} - \boldsymbol{\tau}_s) + \\ & + \left( \frac{7 k}{2 m_s} \nabla T_s - \frac{1}{n_s m_s} \nabla p_s \right) \cdot \boldsymbol{\tau}_s - \frac{e_s}{m_s} \mathbf{q}_s \times \mathbf{B} = \frac{\delta \mathbf{q}_s}{\delta t} \end{aligned}$$

## Photoionization



Photoionization rate  $Q_s(z)$  (i-e pair·cm<sup>-3</sup>·s<sup>-1</sup>)

$n_s(z)$  is the number density of neutral specie, s; (cm<sup>-3</sup>)

$I_\lambda(z)$  is the photon flux at the wavelength  $\lambda$ ; (ph·cm<sup>-2</sup>·s<sup>-1</sup>)

$$Q_s(z) = n_s(z) \sum_{\lambda \leq \lambda_m} I_\lambda(z) \sigma_{s\lambda}^{ion}$$

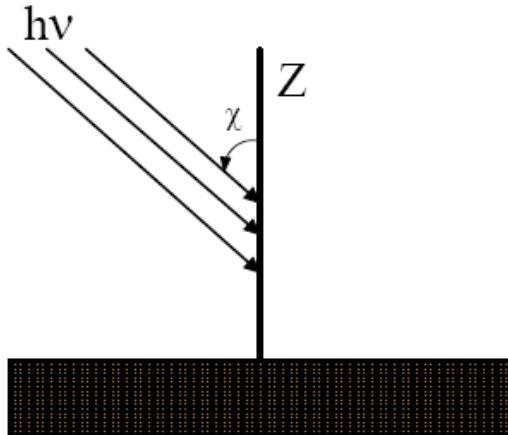
$\sigma_{s\lambda}^{ion}$  is the ionization cross section of the photons with wavelength  $\lambda$  for the neutral specie s; (cm<sup>2</sup>)

$\lambda_m$  is the ionization threshold wavelength

Ionization threshold wavelength

Neutral	$N_2$	$O_2$	$O$
eV	15.58	12.06	13.62
nm	79.58	102.8	91.03

## Absorption of solar radiation into the Earth's atmosphere



$$dI_{\lambda}(z) = -\sum_s I_{\lambda}(z) n_s(z) \sigma_{s\lambda}^a dl$$

$$dl = \sec \chi dz \text{ for } \chi < 80^\circ$$

$$dl = Ch(\chi) dz \text{ for } \chi \geq 80^\circ$$

$Ch(\chi)$  is the Chapman function

$$I_{\lambda}(z, \chi) = I_{\lambda\infty} \exp(-\tau(z, \lambda, \chi))$$

$I_{\lambda\infty}$  is the unattenuated photon flux at the top of atmosphere

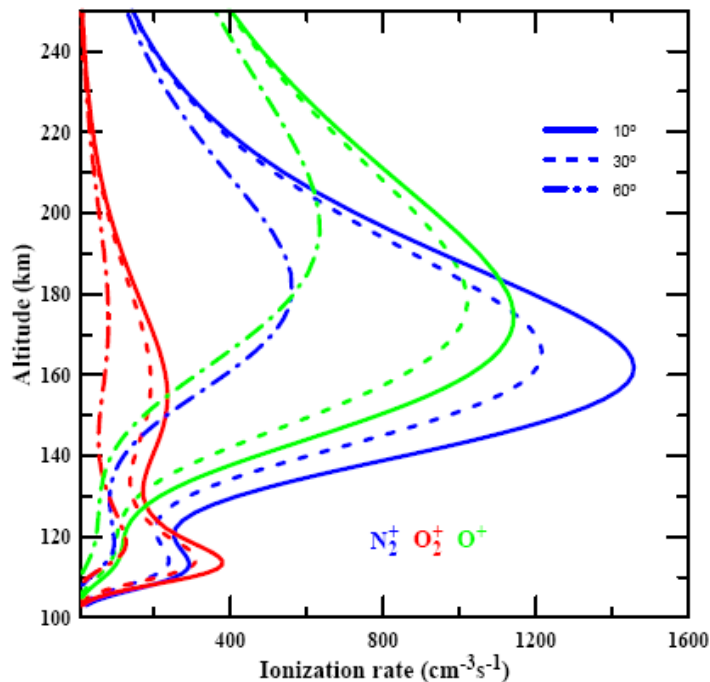
$$\tau(z, \lambda, \chi) \equiv \sec \chi \sum_s \sigma_{sl}^a \int_{\infty}^z n_s(z) dz$$

the optical depth

$$Q_s(z) = n_s(z) \sum_{\lambda \leq \lambda_m} I_{\lambda\infty} \sigma_{s\lambda}^{ion} \exp(-\tau(z, \lambda, \chi))$$

the photoionization rate

## Chapman production function.



$$n_s(z) = n_s(z_0) \exp\left[-\frac{(z-z_0)}{H_s}\right]$$

$$H_s = \frac{kT_s}{m_s g} \text{ - the neutral gas scale height}$$

$$\int_z^{\infty} n_s(z') dz' = n_s(z) H_s$$

$$\tau(z, \lambda, \chi) = \sec \chi \sum_s n_s(z_0) \exp\left[-\frac{(z-z_0)}{H_s}\right] \sigma_{s\lambda}^a H_s$$

For the one-component atmosphere

$$Q(z) = I_{\infty} \sigma^{ion} n(z_0) \exp\left[-\frac{(z-z_0)}{H} - n(z_0) \sigma^a H \sec \chi \exp\left(-\frac{(z-z_0)}{H}\right)\right]$$

Chapman production function

The altitude of the peak production rate

$$z_m = z_0 + H \ln\left[n(z_0) H \sigma^a \sec \chi\right]$$

The maximum production rate

$$Q(z_m, \chi) = \frac{I_{\infty} \sigma^{ion} \cos \chi}{H \exp(1)}$$

# Excitation and ionization by auroral electrons

Kinetic (classical) approach

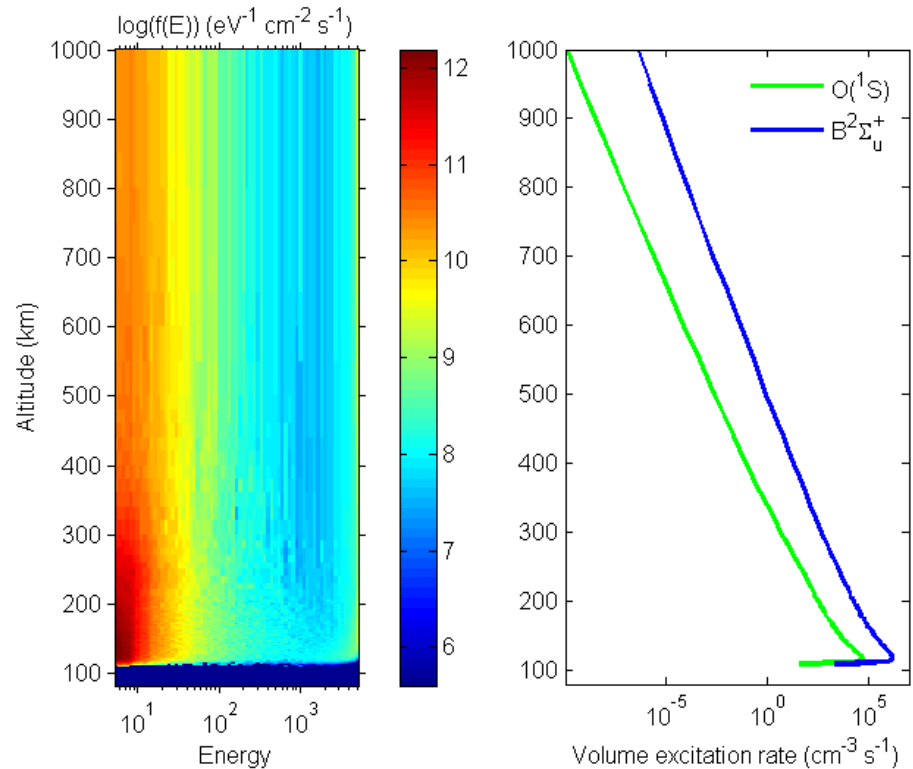
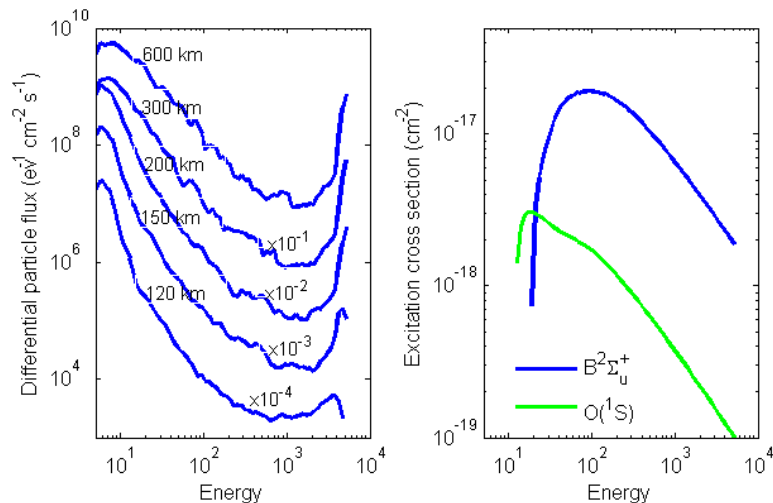
$$Q_i^j(h) = [N_i(h)] \int_E \sigma_i^j(E) f(E, h) dE$$

$Q_i^j$  – excitation rate at altitude  $h$  ( $\text{cm}^{-3} \text{s}^{-1}$ )

$[N_i]$  – concentration of the atmospheric gas ( $\text{cm}^{-3}$ )

$\sigma_i^j(E)$  – excitation cross section of state  $j$  ( $\text{cm}^2$ )

$f(E, h)$  – differential electron particle flux at altitude  $h$  ( $\text{eV}^{-1} \text{cm}^{-2} \text{s}^{-1}$ )



## Auroral electron transport equation

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial s} + \frac{e}{m} \varepsilon_{\parallel} \mu \frac{\partial f}{\partial v} + \left( \frac{e}{m} \varepsilon_{\parallel} - \frac{v^2}{2B} \frac{dB}{ds} \right) \frac{1 - \mu^2}{v} \frac{\partial f}{\partial \mu} = \frac{\delta f}{\delta t}$$

$\mu = \cos \alpha$ ,  $\alpha$  - pitch angle,  $s$  - distance along the magnetic field line

$\varepsilon_{\parallel}$  - externally parallel electric field

$\Phi \equiv \frac{2E}{m^2} f$ ,  $E$  – electron energy

$$\sqrt{\frac{m}{2E}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} + e \varepsilon_{\parallel} E \mu \frac{\partial}{\partial E} \left( \frac{\Phi}{E} \right) + \left( \frac{e \varepsilon_{\parallel}}{E} - \frac{1}{B} \frac{dB}{ds} \right) \frac{1 - \mu^2}{2} \frac{\partial \Phi}{\partial \mu} = \sqrt{\frac{m}{2E}} \frac{\delta \Phi}{\delta t}$$

# Auroral electron transport equation

Steady state, no electric field, constant magnetic field

$$\mu \frac{\partial \Phi(E, s, \mu)}{\partial s} = -a - b + c + d + f + Q$$

$$a = \Phi(E, s, \mu) \sum_k n_k(s) \sigma_k^{tot}(E)$$

Losses of electron with energy E and pitch angle  $\mu$  due to elastic and inelastic collisions

$$b = n_e(s) \frac{\partial(L(E)\Phi)}{\partial E}$$

Energy losses due to collisions with thermal electrons

$$c = \sum_k n_k(s) \sigma_k^{el}(E) \int_{-1}^1 p(E, \mu' \rightarrow \mu) \Phi(E, s, \mu') d\mu'$$

Production of electrons with pitch angle  $\mu$  due to elastic scattering of electrons with pitch angle  $\mu'$

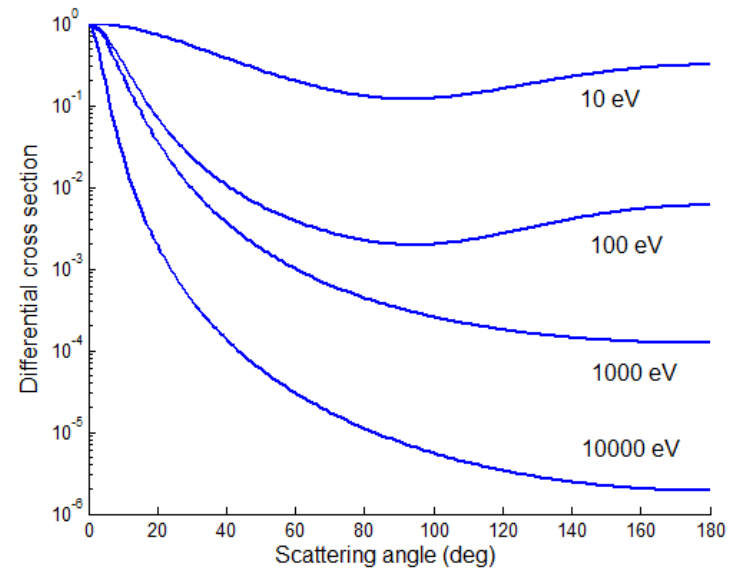
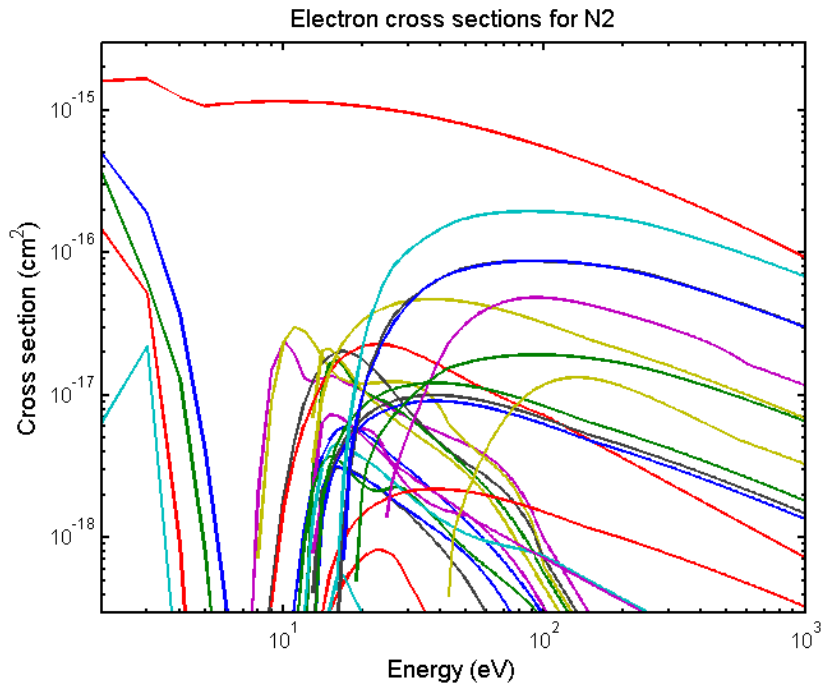
$$d = \sum_k n_k(s) \sum_j \sigma_j^k(E + \Delta E \rightarrow E) \int_{-1}^1 p_j^k(E + \Delta E, \mu' \rightarrow \mu) \Phi(E + \Delta E, s, \mu') d\mu'$$

Production of electrons with energy E and pitch angle  $\mu$  due to inelastic collisions

$$f = \sum_k n_k(s) \int_{E+\Delta E_{ion}}^{\infty} \sigma_{ion}^k(E' \rightarrow E) \int_{-1}^1 p_{ion}^k(E', \mu' \rightarrow \mu) \Phi(E', s, \mu') dE' d\mu'$$

Production of secondary electrons with energy E and pitch angle  $\mu$  in ionization

# Electron collision cross sections



# Excitation and ionization by auroral electrons

## Energy approach

$$W(h) = \rho(h) \int_E \frac{\lambda(E, l/R(E))}{R(E)} f_0(E) dE \quad \text{- electron energy deposition rate at altitude } h \text{ (eV cm}^{-3} \text{ s}^{-1}\text{)}$$

$$Q_i^j(h) = \frac{p_i [N_i]}{\varepsilon_i^j} W(h)$$

$[N_i(h)]$  – gas concentration, ( $\text{cm}^{-3}$ )  
 $i = N_2, O_2, O$

$$p_i = \frac{a_i}{a_{N_2} [N_2] + a_{O_2} [O_2] + a_O [O]}$$

$$a_{N_2} = 1, a_{O_2} = 0.7, a_O = 0.4$$

$\varepsilon_i^j$  - energy cost of  
excited state  $j$ , (eV)

$\rho(h)$  – atmosphere mass density at altitude  
 $h$  ( $\text{gm sm}^{-3}$ )

$\lambda(E, l/R(E))$  – normalized energy dissipation  
function

$R(E)$  – particle range ( $\text{gm cm}^{-2}$ )

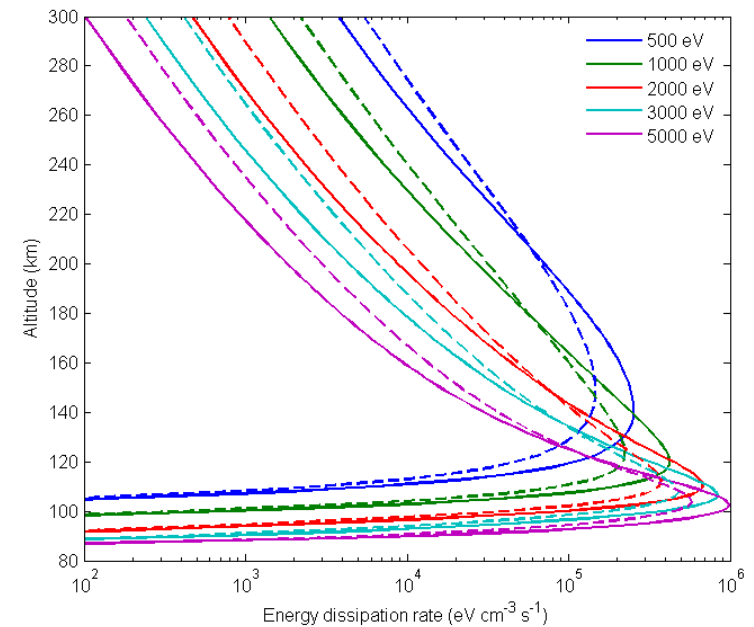
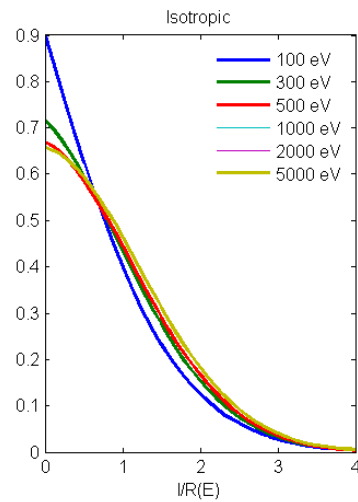
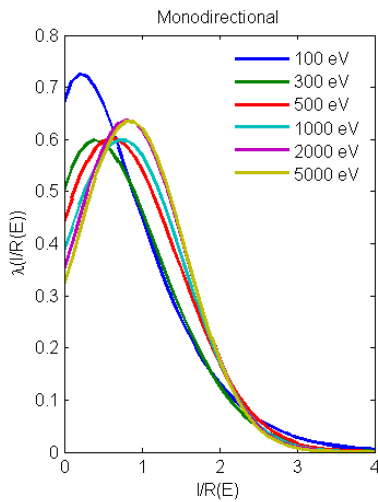
$l = \int_h^{h_0} \rho(h) d(h)$  - distance from the top of  
ionosphere ( $h_0$ ) til altitude  $h$  in  
mass units ( $\text{gm cm}^{-2}$ )

$f_0(E)$  – initial differential electron particle flux at the top of  
ionosphere ( $h_0$ ) ( $\text{eV}^{-1} \text{ cm}^{-2} \text{ s}^{-1}$ )

# Excitation and ionization by auroral electrons

## Energy dissipation rate

$$W(h) = \rho(h) \int_E \frac{\lambda(E, l/R(E))}{R(E)} f_0(E) dE$$



# Excitation and ionization by auroral electrons

## Excitation energy cost

Electron energy lost on excitation of gas  $i$

$$W_i(h) = [N_i(h)] \int_E f(E) \sum_k I_k \sigma^k(E) dE$$

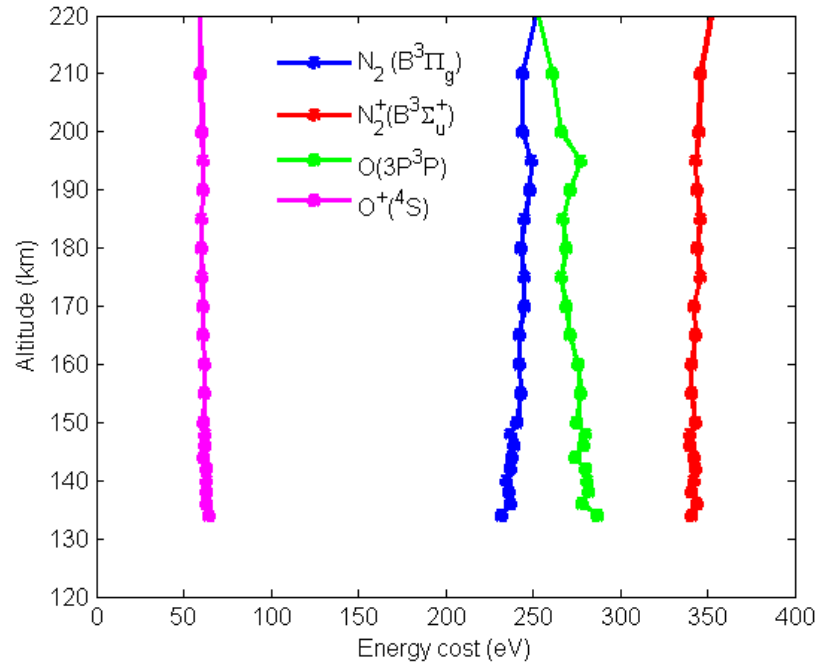
$I_k$  - excitation threshold of  $k^{\text{th}}$  state of gas  $i$

Excitation rate of  $k^{\text{th}}$  state of gas  $i$

$$Q_i^k(h) = [N_i(h)] \int_E f(E) \sigma_i^k(E) dE$$

Excitation energy cost of  $k^{\text{th}}$  state of gas  $i$

$$\varepsilon_i^k(h) = \frac{W_k(h)}{Q_i^k(h)} = \text{const}$$



# Chemical reactions

	Reaction	Rate constant $\text{cm}^3 \text{s}^{-1}$
	Photoionization	
Q1	$O + h\nu \rightarrow O^+ + e$	
Q2	$O_2 + h\nu \rightarrow O_2^+ + e$	
Q3	$N_2 + h\nu \rightarrow N_2^+ + e$	
	Dissociative recombination	
R1	$NO^+ + e \rightarrow N + O$	$4.0 \times 10^{-7} (300/T_e)^{0.5}$
R2	$O_2^+ + e \rightarrow O + O$	$1.95 \times 10^{-7} (300/T_e)^{0.7}$
R3	$N_2^+ + e \rightarrow N + N$	$2.2 \times 10^{-7} (300/T_e)^{0.39}$
	Ion-molecule reactions	
R4	$O^+ + O_2 \rightarrow O_2^+ + O$	$2.1 \times 10^{-11}$
R5	$O^+ + N_2 \rightarrow NO^+ + N$	$1.2 \times 10^{-12}$
R6	$N_2^+ + O_2 \rightarrow O_2^+ + N_2$	$5.0 \times 10^{-11}$
R7	$N_2^+ + O \rightarrow O^+ + N_2$	$9.8 \times 10^{-12}$
R8	$N_2^+ + O \rightarrow NO^+ + N$	$1.3 \times 10^{-10}$

When photochemistry is more important than transport processes, the continuity equation for ions and electron is

$$\frac{dn_s}{dt} = P_s - L_s$$

where  $P_s$  is the production rate and  $L_s$  is the loss rate.

$$\frac{dn(N_2^+)}{dt} = Q_1 - [k_3 n_e + k_6 n(O_2) + (k_7 + k_8) n(O)] n(N_2^+)$$

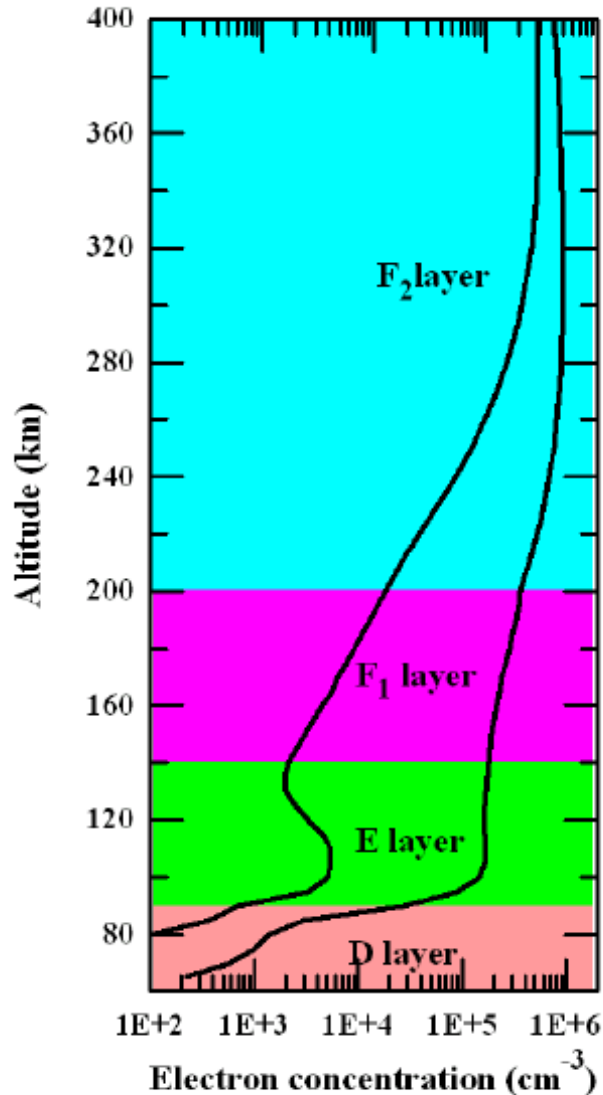
$$\frac{dn(O_2^+)}{dt} = Q_2 + k_4 n(O^+) n(O_2) + k_6 n(N_2^+) n(O_2) - k_2 n_e n(O_2^+)$$

$$\frac{dn(O^+)}{dt} = Q_3 + k_7 n(N_2^+) n(O) - [k_4 n(O_2) + k_5 n(N_2)] n(O^+)$$

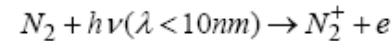
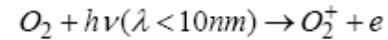
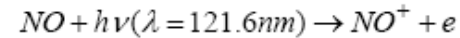
$$\frac{dn(NO^+)}{dt} = k_5 n(O^+) n(N_2) + k_8 n(N_2^+) n(O) - k_1 n_e n(NO^+)$$

$$\frac{dn_e}{dt} = Q_1 + Q_2 + Q_3 - [k_1 n(NO^+) + k_2 n(O_2^+) + k_3 n(N_2^+)] n_e$$

# Earth ionosphere layers



D-layer:

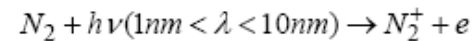
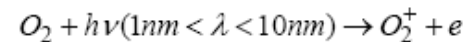
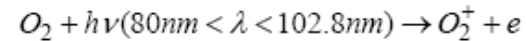


Very complicated ion composition (positive and negative ions, water cluster ions)

Chemical processes dominate.

$$T_e \approx T_i \approx T_n$$

E-layer:

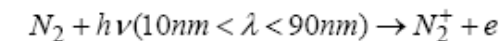
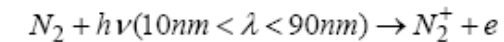


Dominant ions are NO<sup>+</sup>, O<sub>2</sub><sup>+</sup>, N<sub>2</sub><sup>+</sup>

Chemical processes dominate

$$T_e \approx T_i \approx T_n$$

F<sub>1</sub>-layer:

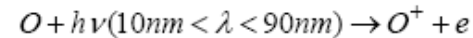


Dominant ions are NO<sup>+</sup>, O<sub>2</sub><sup>+</sup>, N<sub>2</sub><sup>+</sup>

Chemical processes dominate

$$T_e > T_i \approx T_n$$

F<sub>2</sub>-layer:



Dominant ion O<sup>+</sup>

Transition from chemical to diffusion dominance.

$$T_e > T_i > T_n$$

# Quadratic and linear lows of electron loss

E,F<sub>1</sub> regions (90-180 km)

Electron production rate

$$P_e = Q_1 + Q_2 + Q_3 = Q^e$$

Electron loss rate

$$L_e = [k_1 n(NO^+) + k(N_2^+)] n_e$$

For ionospheric plasma  $\sum n(\text{ion}) = n_e$

Effective recombination coefficient

$$\alpha_{\text{eff}} = \left[ k_1 \frac{n(NO^+)}{n_e} + k_2 \frac{n(O_2^+)}{n_e} \right] n_e$$

Quadratic low of the electron losses

$$\frac{dn_e}{dt} = Q^e - \alpha_{\text{eff}} n_e^2$$

F<sub>2</sub> region (>200 km)

Dominant ion is O<sup>+</sup>

$$\frac{dn_e}{dt} = \frac{dn(O^+)}{dt} = Q_1 - L(O^+)$$

O<sup>+</sup> loss rate

$$L(O^+) = [k_4 n(O_2) + k_5 n(N_2)] n(O^+)$$

Effective loss coefficient

$$\beta_{\text{eff}} = k_4 n(O_2) + k_5 n(N_2)$$

Linear low of the electron losses

$$\frac{dn_e}{dt} = Q_1 - \beta_{\text{eff}} n_e$$

$$Q_1 \sim n(O) \sim \exp(-z/H_0)$$

$$\beta_{\text{eff}} \sim n(N_2) \sim \exp(-z/H_{N_2})$$

For steady-state conditions

$$n_e = \frac{Q_1}{\beta_{\text{eff}}} \sim \frac{\exp(-z/H_0)}{\exp(-z/H_{N_2})} = \exp\left[\frac{m_{N_2} - m_O}{kt} g z\right]$$

Electron density increases exponentially with altitude?

Involving the plasma transport is needed.

# Ambipolar diffusion

F<sub>2</sub> region (>200 km)

Major ion is O<sup>+</sup>  $n_i = n(O^+)$

Charge neutrality  $n_e = n_i$

Zero current  $n_e \mathbf{u}_e = n_i \mathbf{u}_i$

Steady state condition  $\frac{\partial \mathbf{u}}{\partial t} \rightarrow 0$

Flow is subsonic  $(\mathbf{u} \cdot \nabla) \mathbf{u} \rightarrow 0$

The ion and electron momentum equations along magnetic field

$$\nabla_{\parallel} p_i - n_i e E_{\parallel} - n_i m_i g_{\parallel} = n_i m_i \nu_{ie} (u_{e\parallel} - u_{i\parallel}) + n_i m_i \nu_{in} (u_{n\parallel} - u_{i\parallel})$$

$$\nabla_{\parallel} p_e - n_e e E_{\parallel} - n_e m_e g_{\parallel} = n_e m_e \nu_{ei} (u_{i\parallel} - u_{e\parallel}) + n_e m_e \nu_{en} (u_{n\parallel} - u_{e\parallel})$$

Using  $n_i m_i \nu_{ie} = n_e m_e \nu_{ei}$ , the addition of the equations yields

$$\nabla_{\parallel} (p_e + p_i) - n_i (m_i + m_e) g_{\parallel} = n_i (m_i \nu_{in} + m_e \nu_{en}) (u_{n\parallel} - u_{i\parallel})$$

Taking into account that  $m_e \ll m_i$  equation reduces to

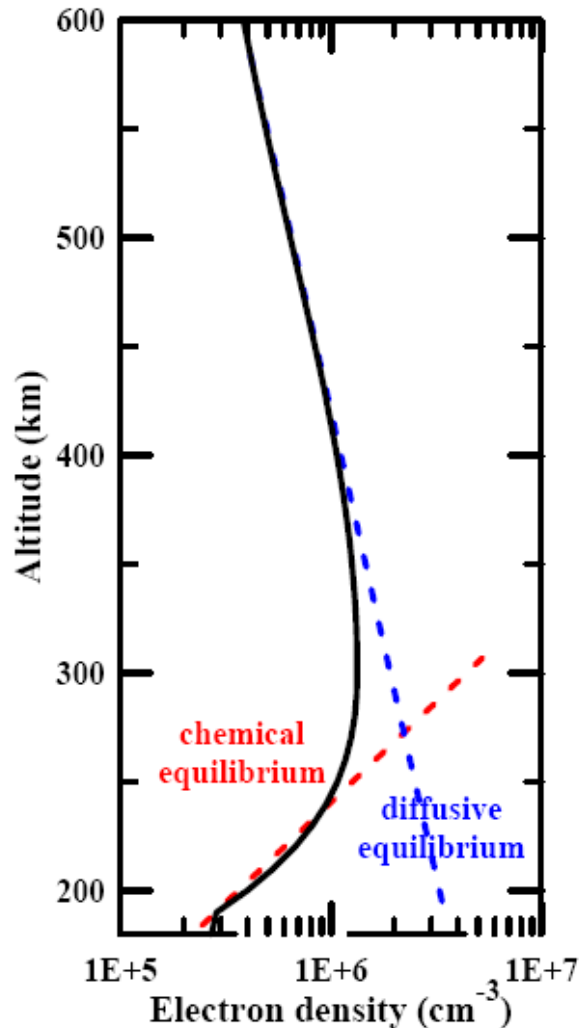
the ambipolar diffusion equation

$$u_{i\parallel} = u_{n\parallel} - D_a \left[ \frac{1}{n_i} \nabla_{\parallel} n_i + \frac{1}{T_p} \nabla T_p - \frac{m_i g_{\parallel}}{2kT_p} \right]$$

$D_a = \frac{2kT_p}{m_i \nu_{in}}$  is the ambipolar diffusion coefficient

$T_p = \frac{T_e + T_i}{2}$  is the plasma temperature.

# Ionospheric maximum formation



Diffusion equilibrium equation

$$\frac{1}{n} \frac{\partial n}{\partial z} = -\frac{1}{H_p} - \frac{1}{T_p} \frac{\partial T_p}{\partial z}$$

For  $T_p = const$

$$n(z) = n_0 \exp\left(-\frac{(z - z_0)}{H_p}\right)$$

Photochemical equilibrium equation

$$n_e(z) = \frac{Q_1}{\beta_{eff}} \sim \exp\left[\frac{m_{N_2} - m_O}{kt} gz\right]$$

The F region peak density occurs at the altitude where the diffusion and chemical processes are of equal importance.

# Auroral aeronomy II

Tima Sergienko

AGF-351

Optical methods in auroral physics research,  
UNIS, Longyearbyen



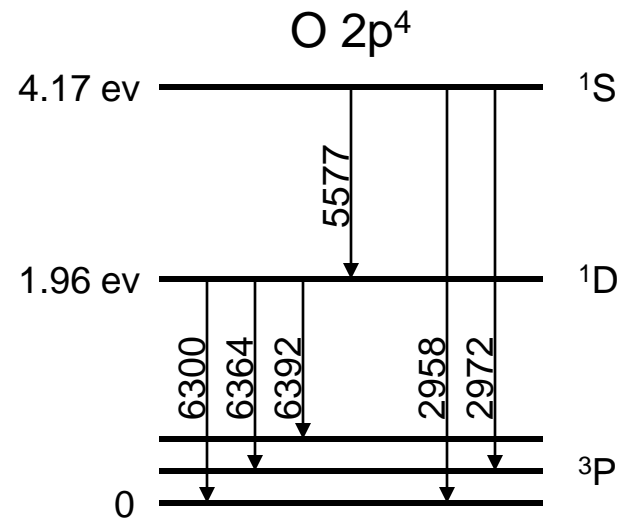
# Volume emission rate and intensity

Atomic line:

$$V_{kk'} = A_{kk'} N_k \quad (\text{cm}^{-3} \text{ s}^{-1})$$

$A_{kk'}$  – Einstein coefficient for transition between states  $k$  and  $k'$  ( $\text{s}^{-1}$ )

$N_k$  – concentration of atoms excited at state  $k$  ( $\text{cm}^{-3}$ )



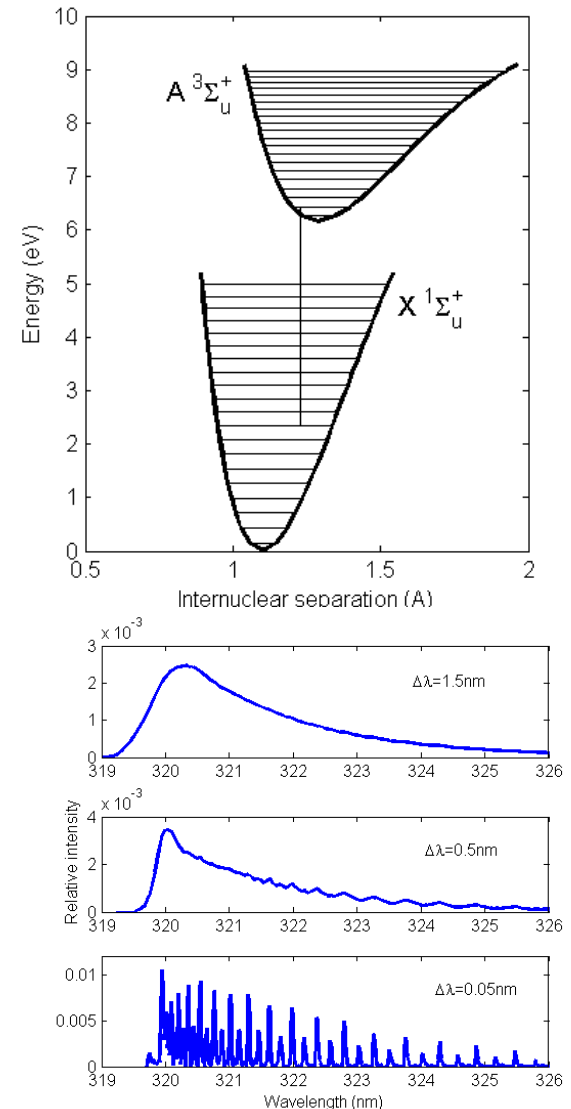
# Volume emission rate and intensity

Molecular band:

$$V_{kk'}^{vv'}(J, J') = \sum_{\substack{J, J' \\ \Delta J=0, \pm 1}} A_{kk'}^{vv'}(J, J') \cdot N_k^v(J)$$

$A_{kk'}^{vv'}(J, J')$  - Einstein coefficient for transition between electronic states  $k$  with vibrational level  $v$  and rotational level  $J$  and electronic states  $k'$  with vibrational level  $v'$  and rotational level  $J'$  (s-1)

$N_k^v(J)$  - concentration of molecular excited at electronic states  $k$  with vibrational level  $v$  and rotational level  $J$  (cm-3)



# Volume emission rate and intensity

Total emission from a column of unit cross section along the line of sight

$$4\pi I = \int_h V(h) dh \quad \text{photon cm}^{-2} \text{ str}^{-1} \text{ s}^{-1}$$

$$I(\text{Rayleigh}) = \frac{4\pi}{10^6} I(\text{photon cm}^{-2} \text{ str}^{-1} \text{ s}^{-1})$$

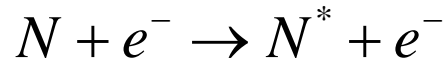
# Volume emission rate and intensity

Continuity equation:

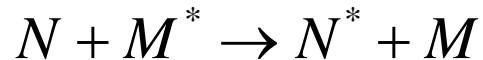
$$\frac{dN^*}{dt} = Q - L$$

Productions,  $Q$

Electron impact



Energy transfer



Radiative cascading

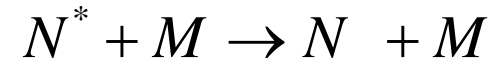


Losses,  $L$

Radiative transfer



Quenching



# Excitation of atmospheric gases by auroral electrons

Kinetic (classical) approach

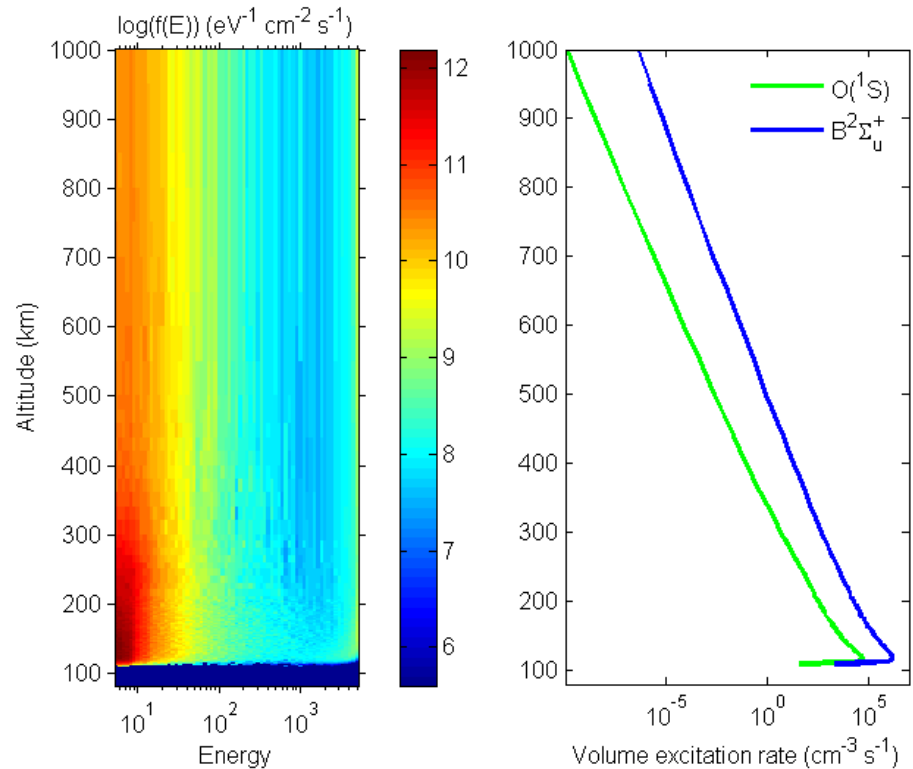
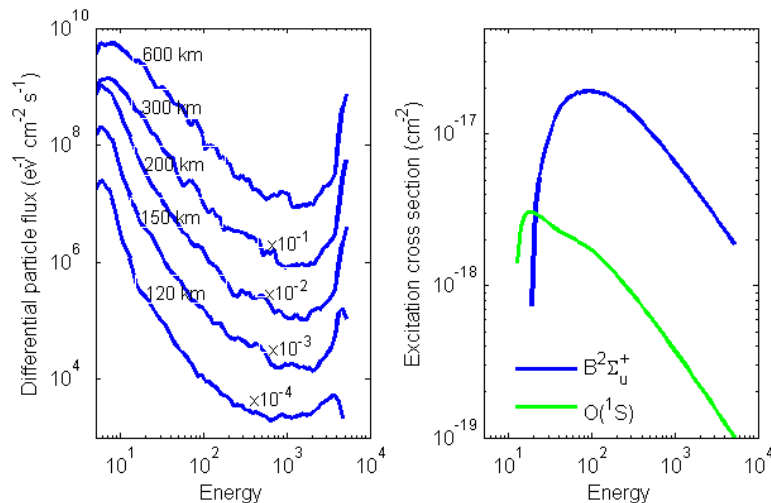
$$Q_i^j(h) = [N_i(h)] \int_E \sigma_i^j(E) f(E, h) dE$$

$Q_i^j$  – excitation rate at altitude  $h$  ( $\text{cm}^{-3} \text{s}^{-1}$ )

$[N_i]$  – concentration of the atmospheric gas ( $\text{cm}^{-3}$ )

$\sigma_i^j(E)$  – excitation cross section of state  $j$  ( $\text{cm}^2$ )

$f(E, h)$  – differential electron particle flux at altitude  $h$  ( $\text{eV}^{-1} \text{cm}^{-2} \text{s}^{-1}$ )



# Auroral emissions

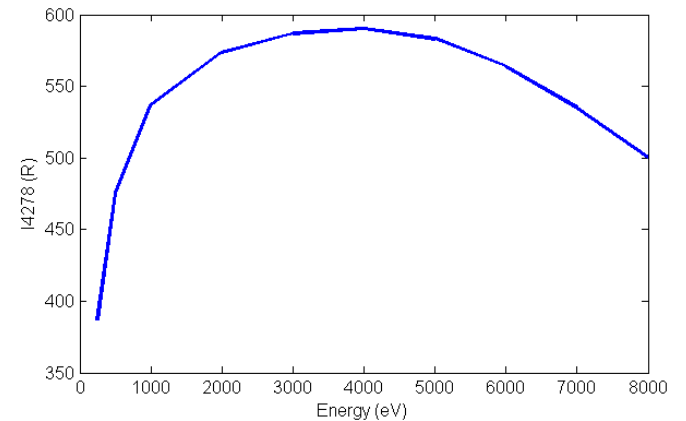
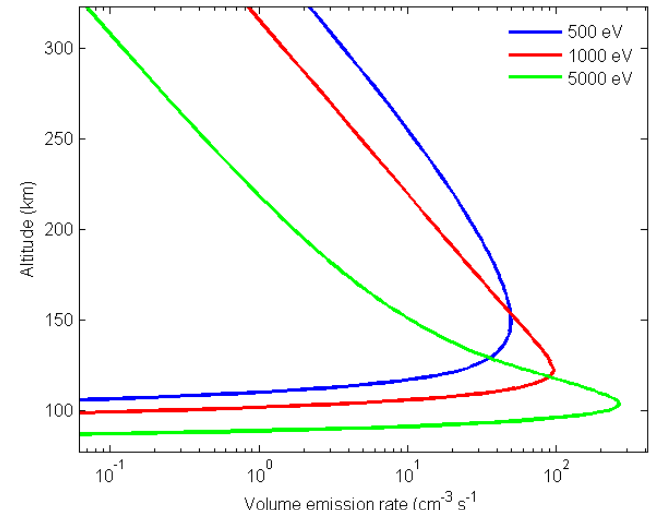
First negative bands of molecular nitrogen ion

$$\frac{dN_v^{B^2\Sigma_u^+}}{dt} = Q - L = 0, \tau < 10^{-5} s$$

$$Q_v^{B^2\Sigma_u^+} = Q^{B^2\Sigma_u^+} q_{v''v}^{X^1\Sigma_g^- \rightarrow B^2\Sigma_u^+} = \frac{p_{N_2} [N_2] W}{\epsilon_{B^2\Sigma_u^+}} q_{v''v}^{X^1\Sigma_g^- \rightarrow B^2\Sigma_u^+}$$

$$L = N_v^{B^2\Sigma_u^+} \sum_{v'} A_{vv'}^{B^2\Sigma_u^+ \rightarrow X^2\Sigma_g^+}$$

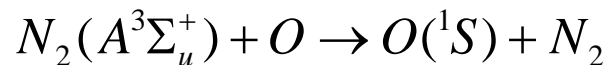
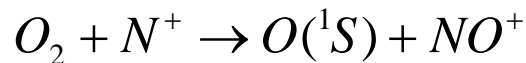
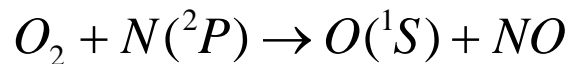
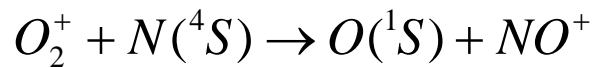
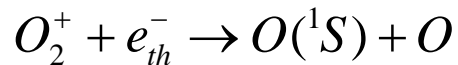
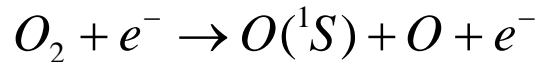
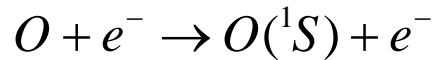
$$V_{vv'} = \frac{q_{v''v}^{X^1\Sigma_g^- \rightarrow B^2\Sigma_u^+} p_{N_2} [N_2] W}{\epsilon_{B^2\Sigma_u^+}} \frac{A_{vv'}^{B^2\Sigma_u^+}}{\sum_{v''} A_{vv''}^{B^2\Sigma_u^+ \rightarrow X^2\Sigma_g^+}}$$



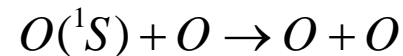
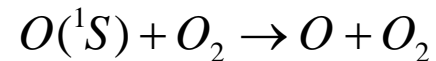
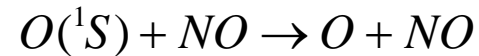
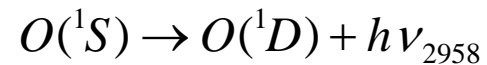
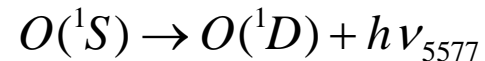
# Auroral emissions

## Auroral green line

### Productions

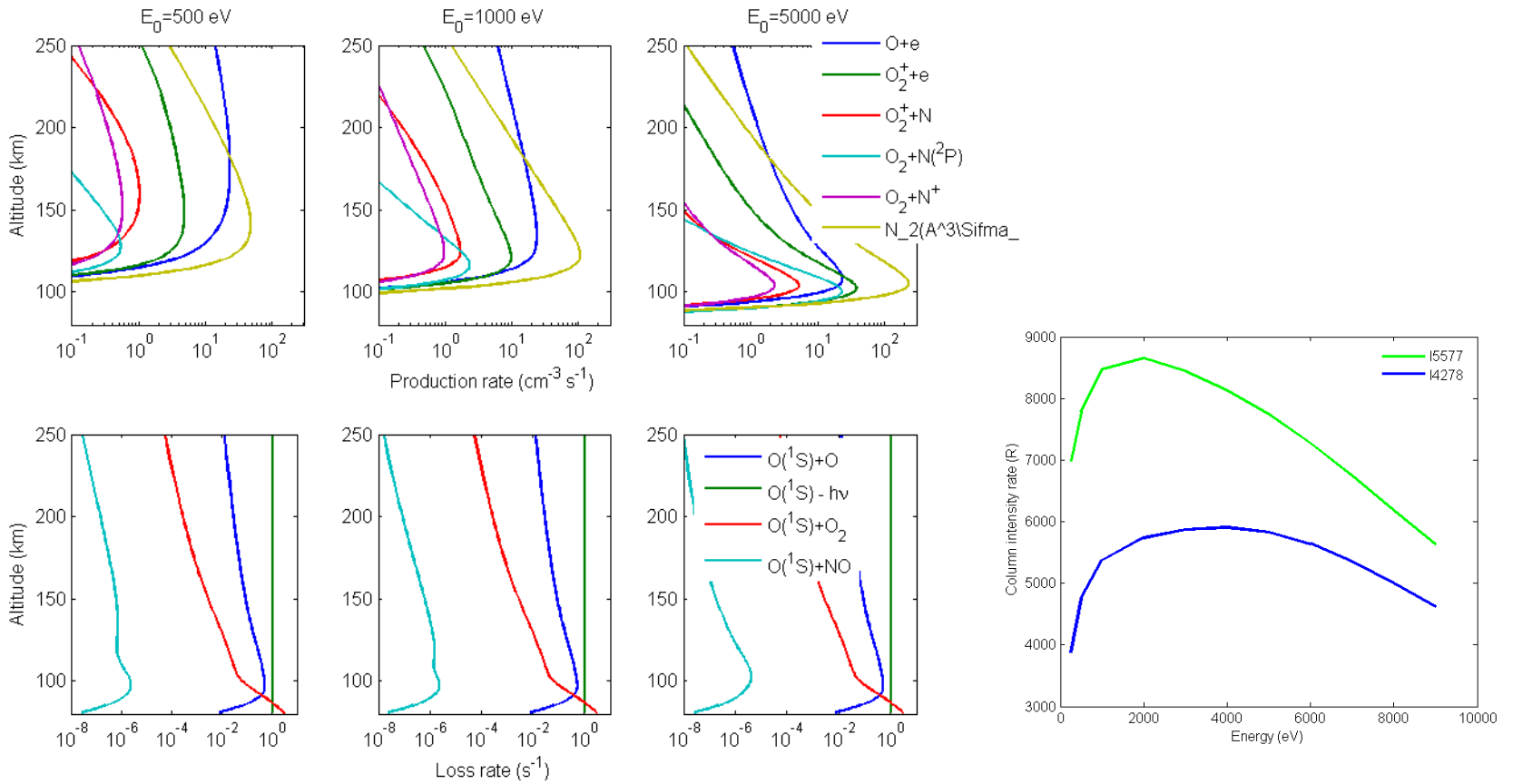


### Losses



# Auroral emissions

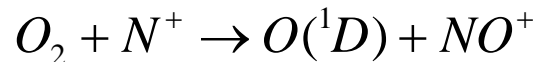
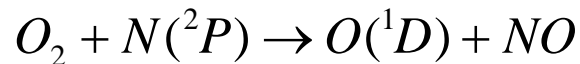
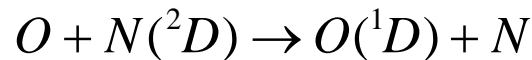
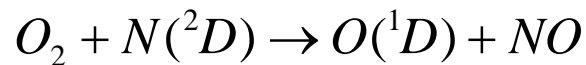
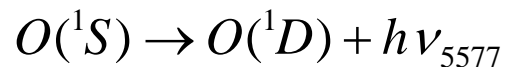
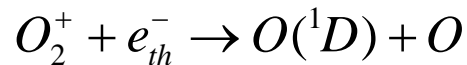
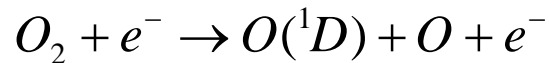
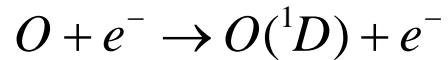
## Auroral green line



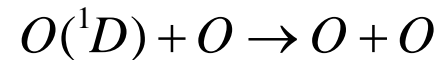
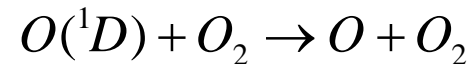
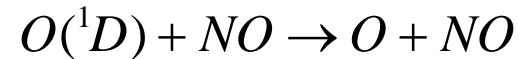
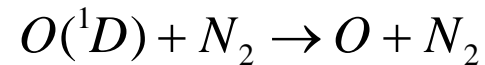
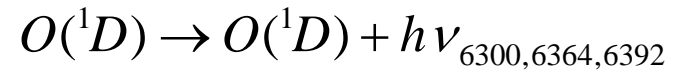
# Auroral emissions

## Auroral red line

### Productions

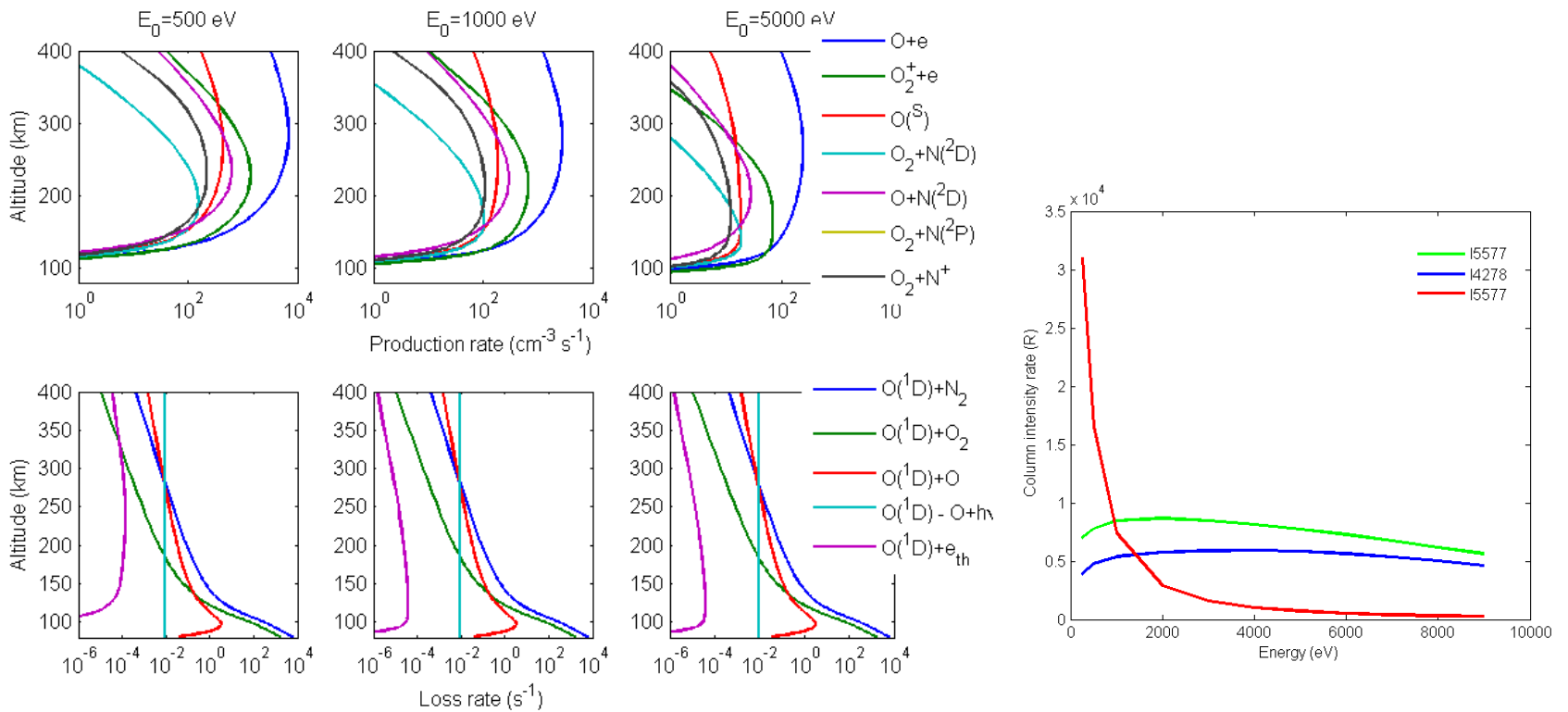


### Losses



# Auroral emissions

## Auroral red line

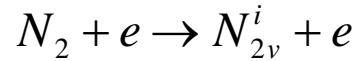


# Auroral emissions

## N2 triple band system

### Production

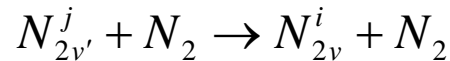
Electron impact



Radiative transfer

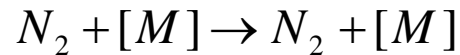


Collisional transfer

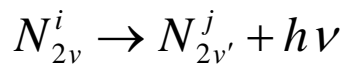


### Loss

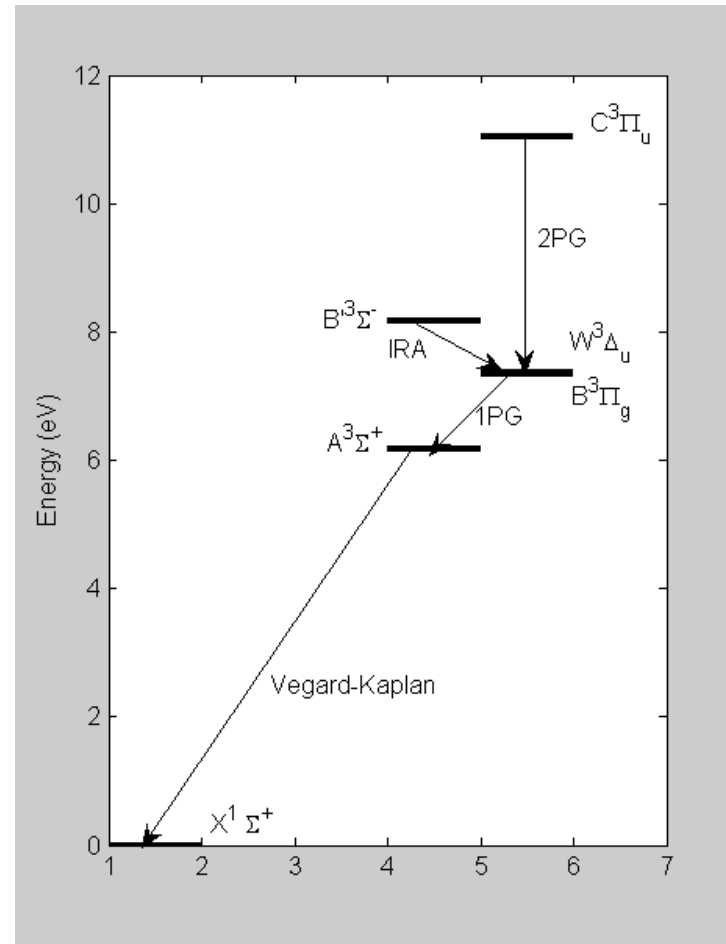
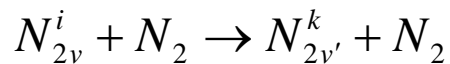
Quenching



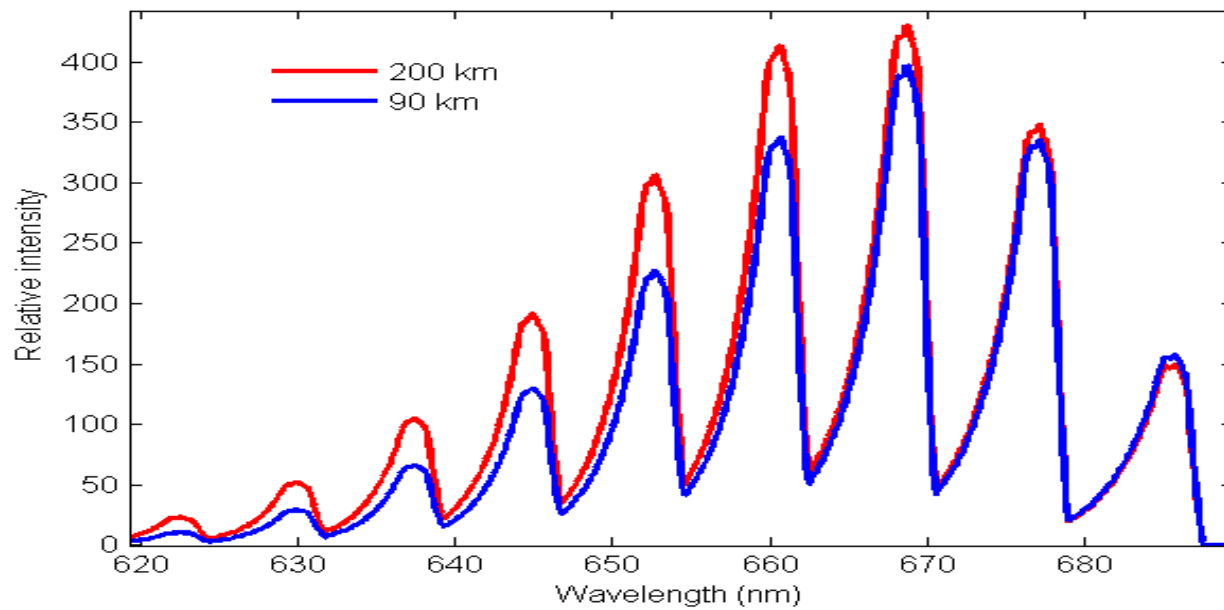
Radiative transfer



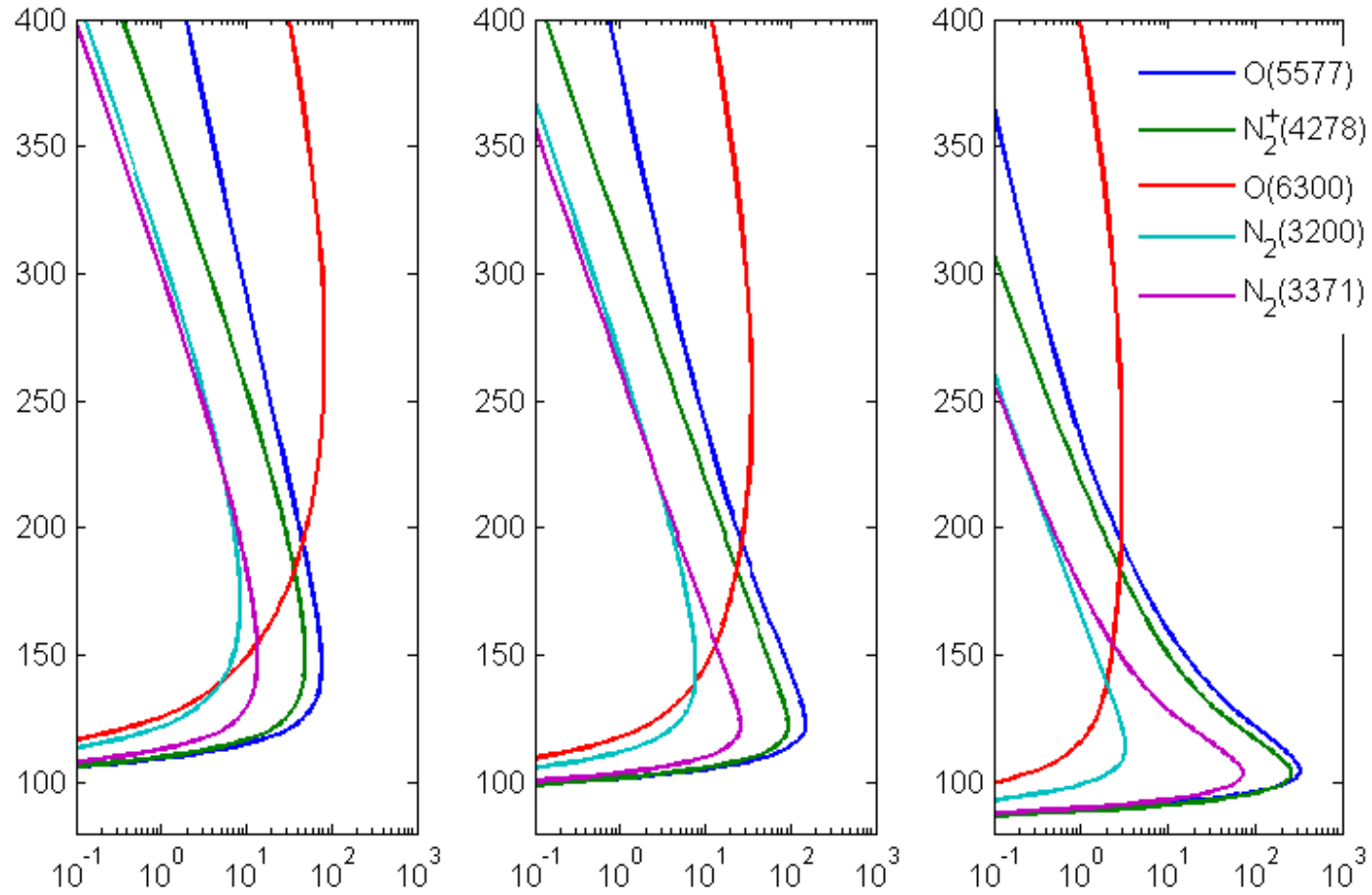
Collisional transfer



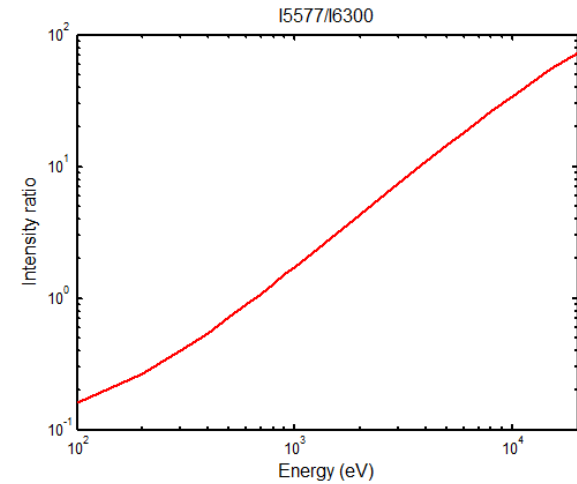
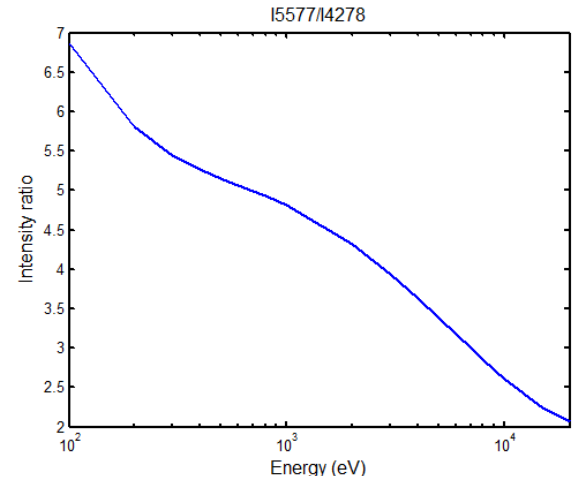
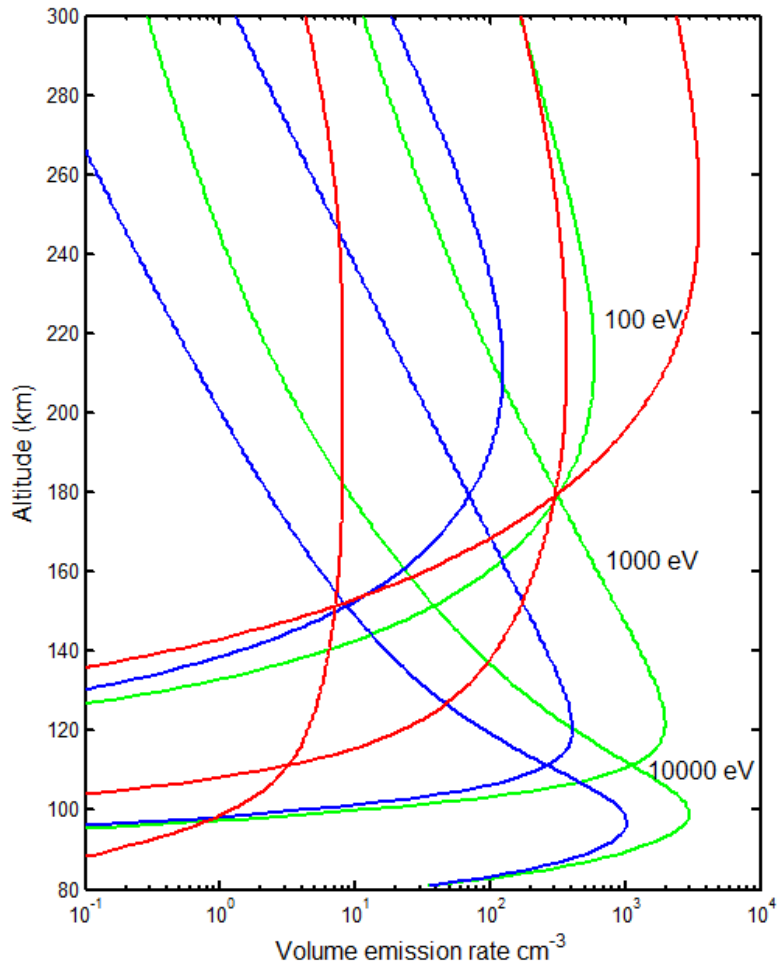
# Auroral emissions



# Auroral emissions



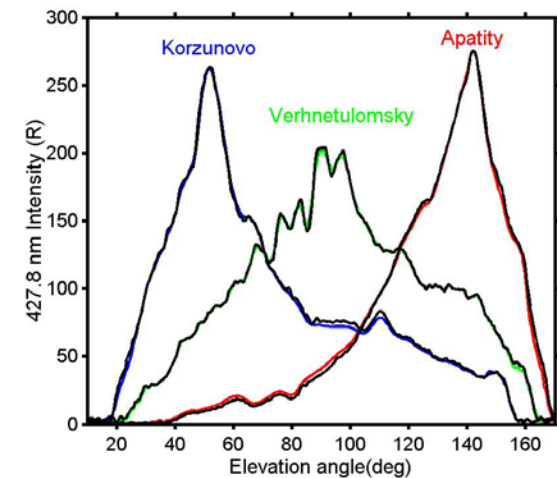
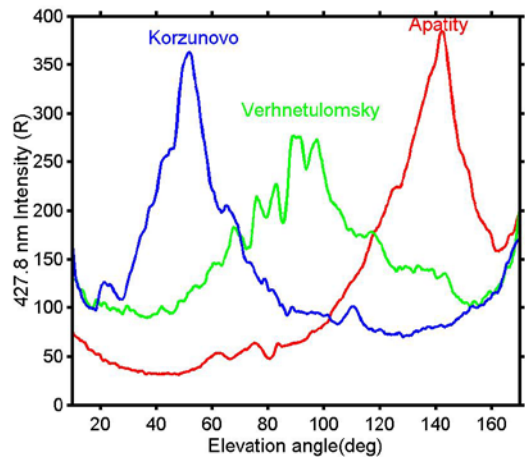
# Diagnostic of the auroral particle parameters



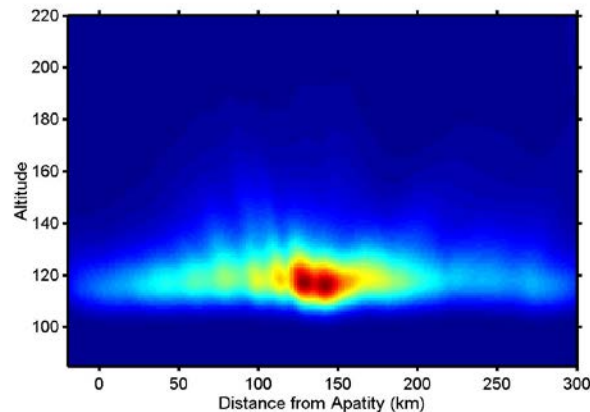
# Reconstruction of electron spectra from auroral emission altitude profile

Scanning photometer records at 23.24:00 UT after background subtraction. Black lines show the photometer records calculated from the reconstructed emissions

Scanning photometer records at 23.24:00 UT

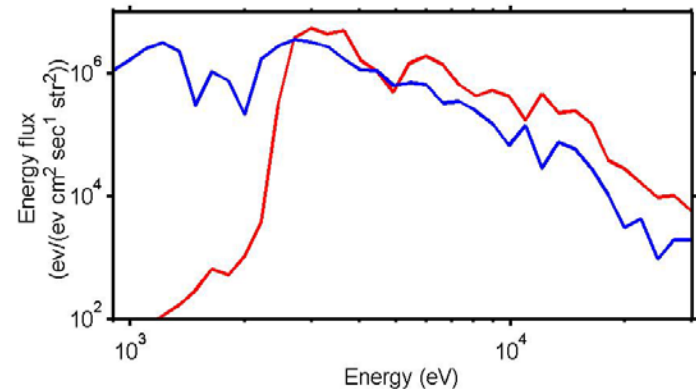
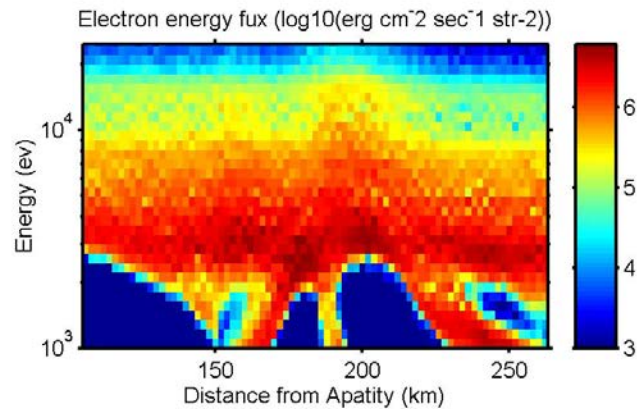
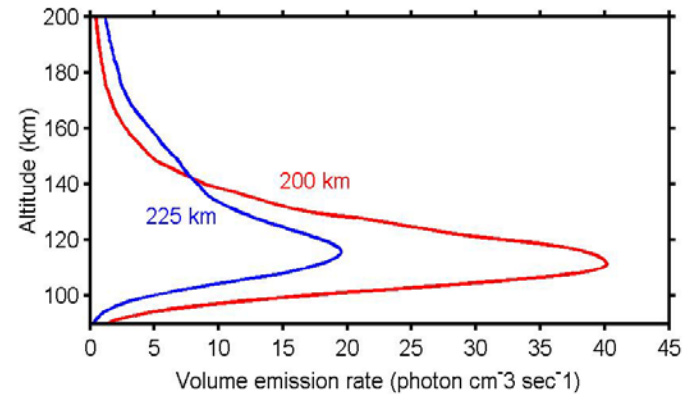
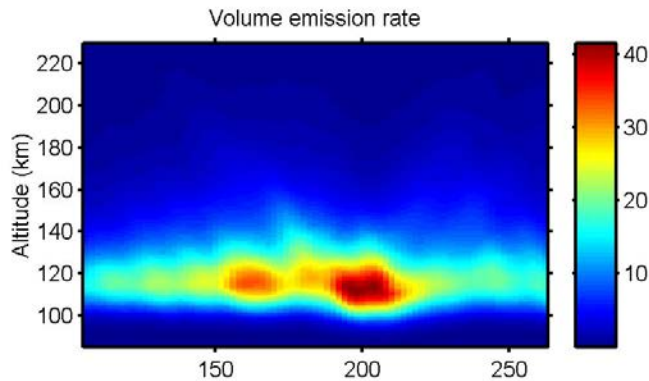


Reconstructed volume emission rate of the 427.8 nm emission



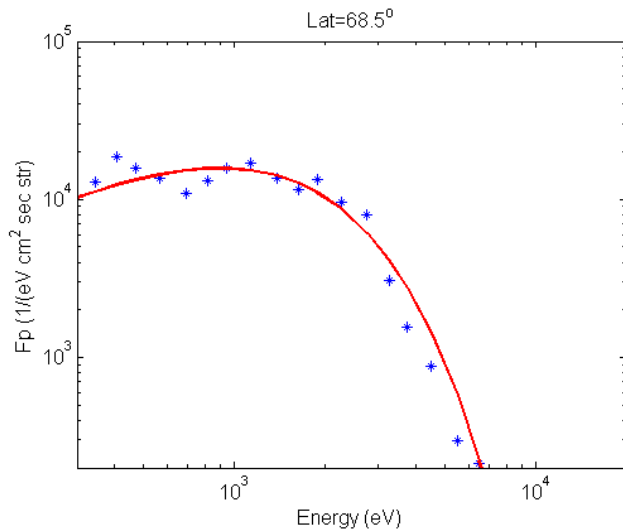
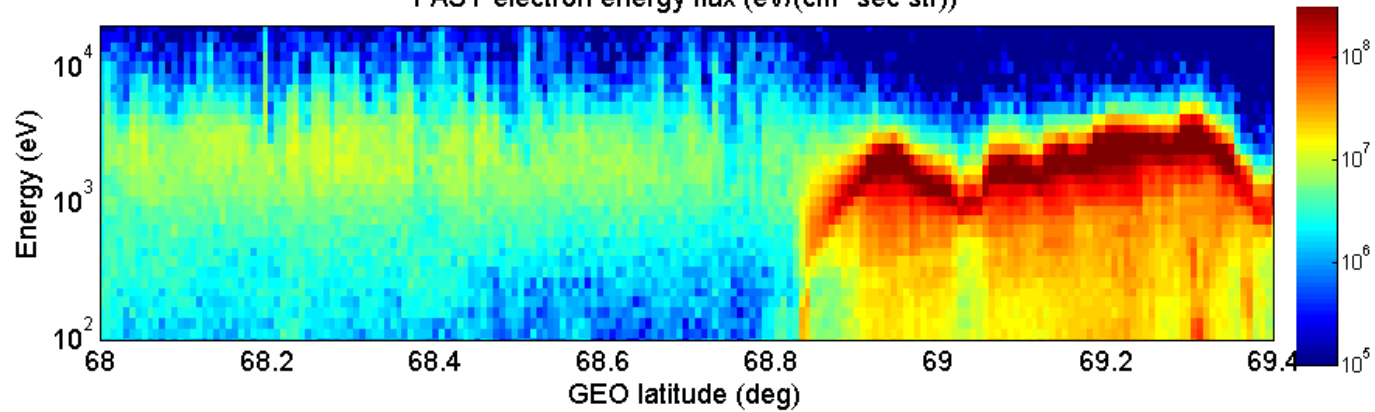
# Reconstruction of electron spectra from auroral emission altitude profile

$$V_{4278} = A_{4278} \frac{\rho(h) p_{N_2} [N_2]}{\mathcal{E}_{v=0}^{B^2\Sigma_u^+}} \int_E \frac{\lambda(E, l/R(E))}{R(E)} f_0(E) dE$$



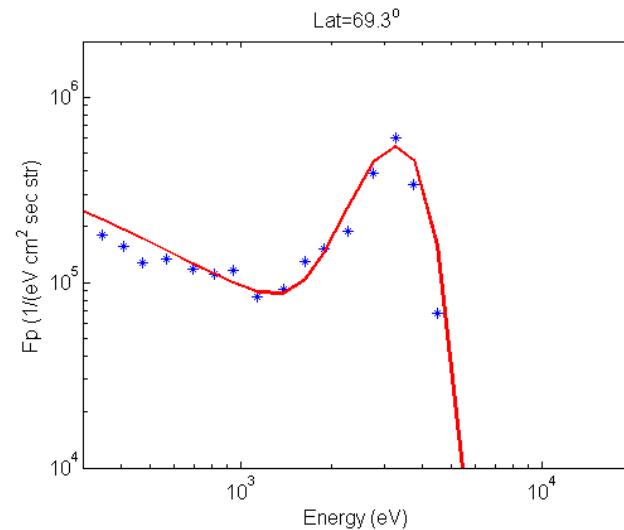
# Auroral electron spectra

FAST electron energy flux (eV/(cm<sup>2</sup> sec str))



Maxwellian

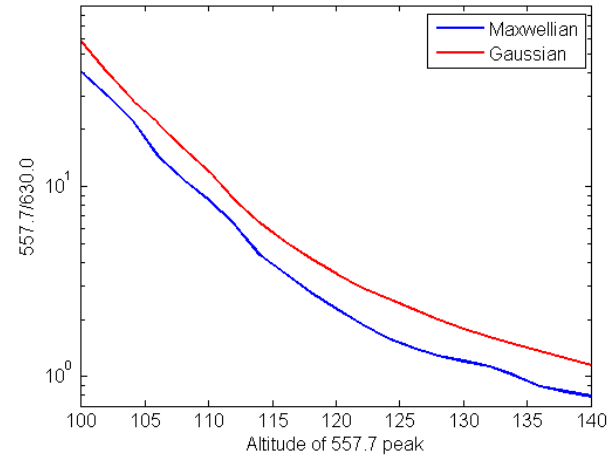
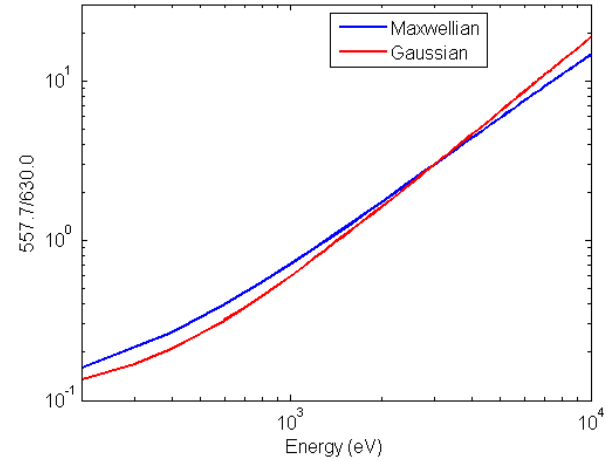
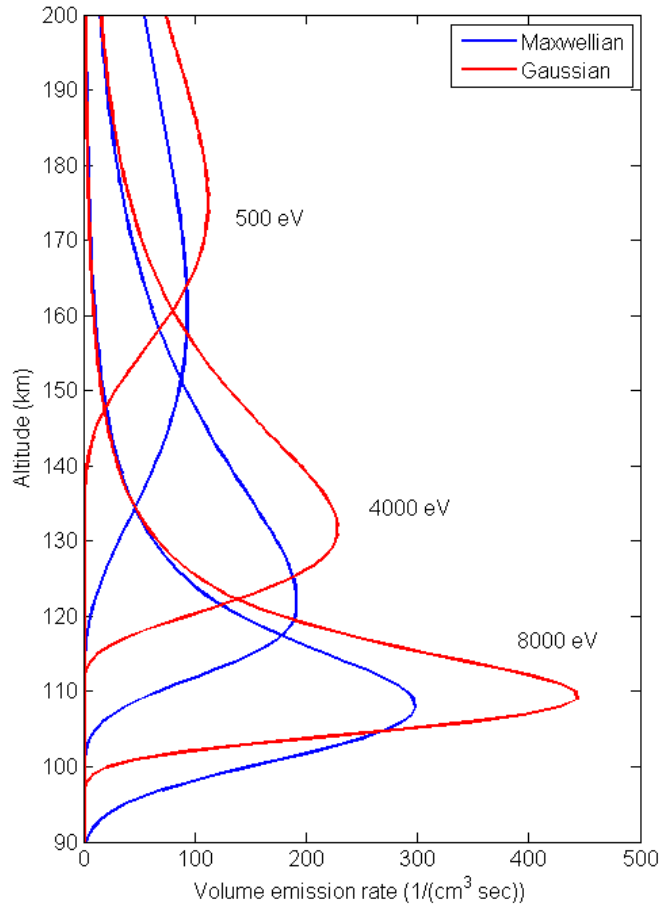
$$F(E)=Q \cdot E \cdot \exp(-E/E_0)$$



Gaussian

$$F(E)=Q \cdot \exp(-(E-E_0)^2/E_0^2)+Q \cdot A/E^b$$

# Maxwellian vs. Gaussian



# Deducing atomic oxygen concentration from Vegard-Kaplan and 1NG emissions

$$0 = Q_{v'} + \sum_{v''} N_{v''}^{B^3} A_{v'',v'}^{1PG} - N_{v'}^{A^3} \left( \sum_{v''} A_{v',v''}^{VK} + \sum_{v''} A_{v',v''}^{1PG} + [N_2]k_{N_2} + [O_2]k_{O_2} + [O]k_O + +[NO]k_{NO} \right)$$

$$[O(h)] = \frac{\frac{I_{v'}^{1NG}(h)}{I_{v'}^{VK}(h)} \frac{\varepsilon_v^{A^3 eff} A_{v',v''}^{VK} \sum_v A_{v',v}^{1NG}}{\varepsilon_{v'}^{B^2} A_{v',v''}^{1NG}} - \sum_v A_{v',v}^{VK} - [O_2]k_{O_2}}{k_O}$$