28. Appendix: Light calibration by C^{14} activated light standards

Light calibration with C^{14} activated light standards

This appendix concerns mainly with the phosphors Y-275 and L-1614 from U.S. Radium Corp. calibrated in Lamberts at the factory and recalculated to Rayleighs as follows:

By definition the light flux is

$$\phi(\text{lumen}) = 680 \int V_\lambda E_\lambda d\lambda$$

where V_λ = standard eye spectral sensitivity

E_λ = the effect in watts/Å

The brightness $B(\text{Lambert}) = \pi \cdot 680 \int V_\lambda E_\lambda d\lambda$

where π is the factor for converting lumens to Lamberts.

The effect/Lambert in watts/Å · Lambert

$$E'_\lambda = \frac{E_\lambda}{\pi \cdot 680 \cdot \int V_\lambda E_\lambda d\lambda}$$

hence in ergs/sec · Å · μL

$$E'_\lambda = \frac{E_\lambda}{\pi \cdot 68 \cdot \int V_\lambda E_\lambda d\lambda} = \frac{\alpha_\lambda}{\pi \cdot 68 \cdot \int V_\lambda \alpha_\lambda d\lambda}$$

where $\alpha_\lambda = E_\lambda / E_{\lambda,max}$

The energy distribution for Y-275 is given in Fig. 1 and for L-1614 in Fig. 2.

With the aid of these curves, the standard eye spectral sensitivity curve, and Simpsons formula for evaluating the value of an integral we get

$E'_\lambda = a_\lambda \cdot 0.685 \times 10^{-5}$ ergs/sec · Å · µL for Y-275

$E'_\lambda = a_\lambda \cdot 1.199 \times 10^{-5}$ "– for L-1614

If E_λ is measured in ergs/cm^2 · ster · sec we have

$E'_\lambda = a_\lambda \cdot 0.685 \times 10^{-5}$ ergs/cm^2 · ster · sec · Å · µL Y-275

$E'_\lambda = a_\lambda \cdot 1.199 \times 10^{-5}$ "– L-1614

converting to rayleighs

$R_\lambda = \frac{a_\lambda \cdot \lambda}{hc} \cdot 4\pi \times 10^{-6} \times 0.685 \times 10^{-5}$ Rayleighs/Å · µL Y-275

$R_\lambda = \frac{a_\lambda \cdot \lambda}{hc} \cdot 4\pi \times 10^{-6} \cdot 0.685 \times 10^{-5}$ "– L-1614

$R_\lambda = \lambda \cdot a_\lambda \cdot 0.433 \times 10^{6}$ Rayleighs/Å · µL Y-275

$R_\lambda = \lambda \cdot a_\lambda \cdot 0.758 \times 10^{6}$ "– L-1614

λ measured in cm

1 rayleigh = $4\pi B$ where B is brightness in units of

10^6 quanta/cm^2 · sec · steradian

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Simpsons formula

$$\int_a^b f(x)dx = \frac{h}{3}(f_o + 4U + 2J + f_n)$$

where $U = f_1 + f_3 + \cdots f_{n-1}$

$J = f_2 + f_4 + \cdots f_{n-2}$

h = the step (here = 100 Å)

If the values f_n are exact Simpsons formula gives the value of the integral with an accuracy of 0.14 %. But the values $f_n = a_\lambda \cdot V_\lambda$

have in fact an uncertainty of $|\Delta f_n| = a_\lambda|\Delta V_\lambda| + V_\lambda|\Delta a_\lambda$

where $\Delta V_\lambda = \Delta a_\lambda = 0.5 \times 10^{-3}$, which gives an accuracy of 0.35 %.

The total accuracy will be 0.5 %.

The energy distribution a_λ for Y-275, L-1614 and the sensitivity curve of the eye V_λ.

Wavelength Å	Y-275 a_λ	L-1614 a_λ	Y-275 V_λ	L-1614 V_λ
4000		0.070		0.0004
100		0.089		0.0012
200		0.120		0.0040
300		0.158		0.0116
400	0.010	0.213	0.023	0.023
500	0.015	0.270	0.038	0.038
600	0.028	0.344	0.060	0.060
700	0.043	0.500	0.091	0.091
800	0.068	0.790	0.139	0.139
900	0.102	0.946	0.208	0.208
5000	0.154	1.000	0.323	0.323
100	0.182	0.962	0.503	0.503
200	0.300	0.827	0.710	0.710
300	0.480	0.615	0.862	0.862
400	0.655	0.449	0.954	0.954
500	0.706	0.337	0.995	0.995
600	0.722	0.254	0.995	0.995
700	0.790	0.198	0.952	0.952
800	0.912	0.151	0.870	0.870
900	0.988	0.119	0.757	0.757
6000	0.987	0.096	0.631	0.631
100	0.898	0.080	0.503	0.503
200	0.763	0.071	0.381	0.381
300	0.712	0.066	0.265	0.265
400	0.555	0.066	0.175	0.175
500	0.350	0.066	0.107	0.107
600	0.200	0.066	0.061	0.061
700	0.090	0.066	0.032	0.032

We get the following values for R_λ in Rayleighs/Å μL for phosphor Y-275 and L-1614

Wavelength Å	Y-275 R_λ/Å μL	L-1614 R_λ/Å μL
4000		2.12
100		2.77
200		3.82
300		5.15
400	0.19	7.10
500	0.29	9.21
600	0.56	12.00
700	0.86	17.8
800	1.41	28.7
900	2.16	35.1
5000	3.33	37.9
100	4.02	37.2
200	6.75	34.4
300	11.02	24.7
400	15.32	18.4
500	16.81	14.1
600	17.51	10.8
700	19.5	8.56
800	22.9	6.64
900	25.24	5.32
6000	25.64	4.37
100	23.72	3.70
200	20.48	3.34
300	19.42	3.15
400	15.38	3.20
500	9.85	3.25
600	5.72	3.30
700	2.61	

Example: At 5600 Å we get 17.51 R/Å μL
The brightness of the standard is 20 μL. Hence the standard gives 350.2 R/Å at 5600 Å.